ACKNOWLEDGEMENTS

This report draws substantially on my Ph.D. research which was supervised by Professor John O' Donoghue at the University of Limerick and completed in November 2008. Anyone wishing to read more detailed information about the study can read the dissertation which is available from me on request.

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PREFACE

This report/monograph is based on a doctoral study completed by Dr Máire Ní Ríordáin at the National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL). We are charged under our brief at the National Centre to engage in evidence-based world class research in Science and Mathematics Teaching and Learning and to bring our findings to bear, *inter alia*, on advice offered to stakeholders in Irish education. This is the second report in the series following NCE's groundbreaking report, *Out-of-field teaching in Post-Primary Mathematics Education: An analysis of the Irish context* (2009) and it focuses on language issues in mathematics teaching and learning.

The study focuses on the plight of a significant but growing minority of Irish students studying mathematics in a bilingual setting (c. 7% of the national cohort of primary and 2.5% of post-primary students in 2007). It is the first study to examine whether learning issues arise in mathematics classrooms for students and teachers at the transition from Gaeilge medium to English medium education. This report demonstrates that priorities in science and mathematics education intersect national priorities in language policy, which is timely in the context of a national focus on literacy and numeracy issues.

The author applies and exploits international research on bilingual education generally and bilingual mathematics education in particular to ground her study and advance her thesis. Professor Bill Barton, an eminent international authority in the field and her external examiner, concludes that 'the study is both timely and also represents work at the forefront of research in bilingual mathematics education'. This is underlined by her papers in *Educational Studies in Mathematics* and the *Mathematics Education Research Journal*.

The report throws light on previously unnoticed and unreported learning issues in mathematics for Gaeilgeoirí (students who learn through the medium of Irish) who make the transition from Gaeilge medium education to English medium education, and demonstrates the advantages of a bilingual approach to mathematics education in Irish schools. The report adds new evidence-based analyses on language issues in

mathematics education that are timely in terms of the national Irish language policy debate, formulation and implementation. A major finding of the study points to the advantages of bilingualism for mathematics education. Also the awareness that the report brings in terms of equity issues offers the opportunity for these to be addressed in the context of new language policy development.

The Directors are pleased to discharge their brief to advise on matters related to Science and Mathematics teaching in this way and commend this report to all who have a stake in Irish education and particularly to those front-line agencies involved in improving matters in Mathematics and Science teaching at all levels.

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SUMMARY OF THE STUDY

Introduction

Teaching and learning through the medium of Gaeilge (Irish) is the natural school environment for teachers and pupils in the Gaeltacht areas (Irish speaking districts) of Ireland. A parents' initiated movement in the early seventies was responsible for the establishment of Gaeilge-medium primary and secondary schools outside of the Gaeltacht regions, known respectively as Gaelscoileanna and Gaelcholáistí (Coady & Ó Laoire, 2002). The number of students attending Gaeilge-medium schools has witnessed a marked increase (Fás ar an nGaelscolaíocht sa Ghalltacht, 2006). However, learning issues may arise both for students and teachers at the transition from Gaeilge-medium education to English-medium education. This is inclusive of both key transition periods – primary level to second level education, and second level to third level/further education. Gaeilgeoirí submerged in the transitions under investigation will be required not only to learn new mathematics, but also to learn mathematics through the medium of English (Barwell, 2003). This research report will present findings from a doctoral study undertaken on bilingualism and mathematics education in the Irish context, the first study of its type to examine the Irish situation.

Aim of the Study

The aim of the study was to investigate Irish post-primary and undergraduate bilingual mathematics students' (Gaeilgeoirí) experiences of learning mathematics through the medium of English, their second language of learning. This includes examining the relationship between language proficiency and performance on mathematics word problems; investigating additive and subtractive bilingualism in an Irish context; examining sources of difficulty encountered with the English language mathematics register; and in the process developing significant insights into cultural and pedagogical influences on the transition from Gaeilge-medium to English-medium mathematics education.

Additive and Subtractive Bilingualism

There are several different types of bilingual education programmes and within these types further subdivisions arise. These programmes can be classified under the broad terms of weak or strong bilingual education (Baker, 2000). Weak forms tend to include schools/institutions that contain bilingual individuals as opposed to encouraging bilingualism. These schools/institutions usually enrol language minority students with the aim of developing learning through the majority language (Baker & Prys Jones, 1998). On the other hand, strong forms of bilingual education are developed when the primary aim is to develop complete bilingualism in both languages, and both cultures are supported (Baker & Prys Jones, 1998). Furthermore the concept of weak and strong bilingual education programmes is associated with subtractive and additive bilingualism. Subtractive bilingualism usually occurs when a majority language replaces a minority language (Lambert, 1990). Students are forced to adapt quickly into mainstream education where the majority language is used as the medium of instruction. No support is given to the mother-tongue, resulting in it slowly being replaced (Bournot-Trites & Tellowitz, 2002). This is similar to the experiences of Gaeilgeoirí transferring from Gaeltacht schools (Maintenance Heritage Language) to all-English-medium schools. The situation is different in the case of additive bilingualism. In this instance the mother-tongue of the child is the majority language and they opt to study through a second language, which is a minority language (Lambert, 1990). This is similar to Immersion education (Gaelscoileanna/Gaelcholáistí) in operation in Ireland. The intention is not to replace the majority language but to develop the second language (Bournot-Trites & Tellowitz, 2002).

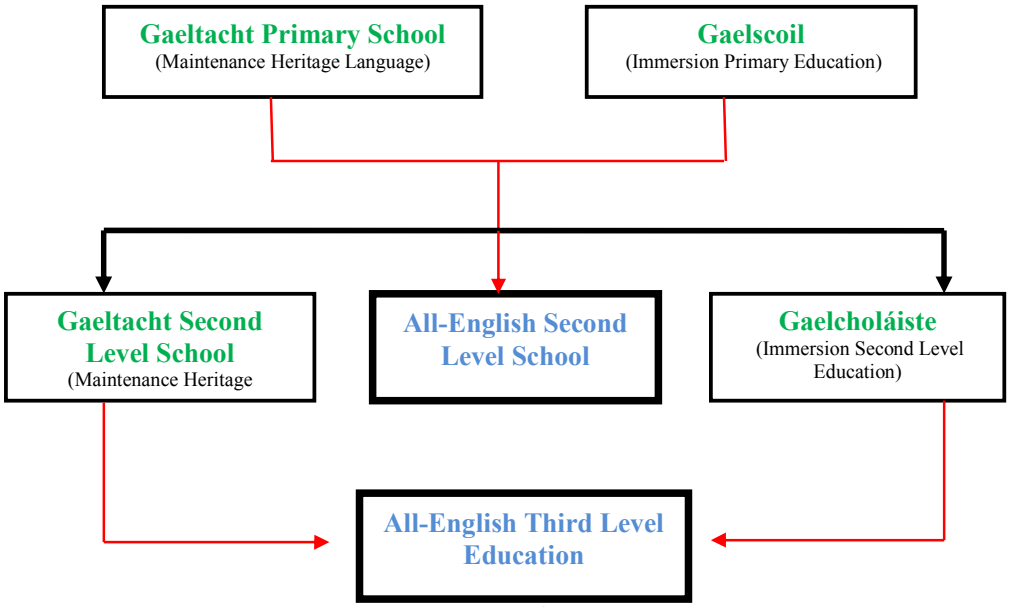
Description of the Study

The study incorporated three phases and it was carried out over the course of a three year period (October 2005 – September 2008). A mixed methods approach was adopted in the study (qualitative and quantitative data/analysis). In ***Phase 1*** a comprehensive review of literature was carried out in order to gain an in-depth knowledge of the area of learning mathematics through the medium of a second language, as well as examining areas such as psycholinguistic theories, bilingualism

and bilingual education. This Phase incorporated and combined conclusions and recommendations from an undergraduate dissertation with new exploratory research. Phase 1 facilitated the design of the methodology to be employed in Phase 2 of this research project. ***Phase 2*** was devoted to data collection at both transitions, from primary to second level and from second level to third level education. The qualitative and quantitative analysis of the data collected was carried out in ***Phase 3*** and relevant findings discussed.

Subjects Involved in the Study

This study was concerned with Gaeilgeoirí in the transition from Gaeilge-medium (primary or second level education) to English-medium mathematics education (second or third level education). Two types of Gaeilge-medium education exists at primary and secondary level in Ireland (Gaeltacht schools and Immersion Education), it is anticipated that the previous learning environments of the students entering the submersion contexts may be of significance. Thus it was necessary to investigate both types of Gaeilge-medium education and to establish if a relationship exists between the type of Gaeilge-medium education attended and the students' achievement in mathematics in an English-medium education context. Monolingual English students were sourced at each transition in order to facilitate a comparison between groups. The situation is described in the following diagram.



Key Findings

The significant overall conclusions emerging from the research undertaken include:

- Language proficiency and performance on mathematical word problems are related for Gaeilgeoirí (Cummins' Threshold Hypothesis, 1976).
- Support was found for Immersion Education and its influence on the development of Additive bilingualism.
- Maintenance Heritage Language education may contribute to Gaeilgeoirí experiencing Subtractive bilingualism on entering English-medium mathematics education.
- Learning mathematics through the medium of Gaeilge at primary level education may enhance long-term mathematical understanding and attainment in English-medium second level education.
- Gaeilgeoirí in the transition from Gaeilge-medium primary level education to English-medium second level mathematics education experience a disadvantage of 8.7 percent in performance when assessed through English.
- When engaged in mathematical word problem solving (through the medium of English) Gaeilgeoirí's understanding tends to break down at the comprehension stage.
- Sources of difficulty encountered with the English mathematics register include syntax, semantics and mathematics vocabulary.
- Gaeilgeoirí employ both languages when engaged in mathematical problem solving.
- There is a clear lack of awareness of bilingualism by Gaeilgeoirí.

Recommendations for Mathematics Teachers in English-medium Schools

The focus of this investigation has been on Gaeilgeoirí in the transition from Gaeilge-medium to English-medium mathematics education. Clearly the teacher is going to play a significant role in facilitating this transition. Mathematics teachers need to:

- Make mathematics accessible.
- Teach the language of mathematics.
- Create language supportive environments.
- Connect the mathematics content to the students' background and experiences.
- Vary instructional methods.
- Use authentic and meaningful assessment methods.

Recommendations for Mathematics Teachers in Gaeilge-medium Schools

Teachers involved in Gaeilge-medium primary and second level education can incorporate some aspects into their mathematics teaching so as to ease the transition to English-medium mathematics education for Gaeilgeoirí. Teachers in Gaeilge-medium schools should:

- Ensure that their students are aware that they will be transferring to a new language of learning, either at second or third level education.
- Introduce some partial instruction through the medium of English in the later years of primary and second level schooling.
- Assess upper primary and second level students' language proficiencies and mathematics performance.

Recommendations in relation to the Gaeilge Language

The research has demonstrated that the Gaeilge language and previous learning through the medium of Gaeilge is of importance to Gaeilgeoirí's mathematical learning through the medium of English. The author does not claim to be a linguistic expert but there are two key recommendations in relation to Gaeilge that she would like to pursue further and will provide additional significant insights into the Irish context. These are:

- 'Code switching' or language switching is occurring during mathematical problem solving for Gaeilgeoirí. Further research needs to look at this aspect and how it can be incorporated into the mathematics classroom.

- There are cognitive benefits for learning mathematics through the medium of Gaeilge. Future Government initiatives may consider the option of providing students with the opportunity of studying particular subjects through the medium of Gaeilge at primary and post-primary level in English-medium schools.

Proposal for a Bilingual Primary and Second Level Education System

There are many benefits to be gained from becoming bilingual in both languages, as opposed to dominance in one language. This in particular is evident from studies undertaken in Canada and Wales in which successful Immersion bilingual education programmes are in operation and students are reaping the cognitive benefits from these programmes (Cummins & Swain, 1986; Swain, 1996; Williams, 2002). Therefore, the author recommends the development of language policies that place an emphasis on the development of *both* Gaeilge and English so that students can become bilingual and thus reap the cognitive benefits of this.

Description of Terms used in the Study

The following is a list of terms used in the monograph with an explanation of how they have been used. The word in *italics* is the term, with the description given directly across from it.

Additive Bilingualism	Occurs when a second language and culture have been acquired without loss or displacement of an individual's first language and culture. Positive self-concept usually developed.
Bilingual	Having two distinct languages, Gaeilge and English in this case.

Gaeltacht	District/area in which Gaeilge is the language of the community and the schools. There are seven Gaeltachtaí in total in Ireland.
Gaelscoileanna	Gaeilge-medium primary level schools located outside of the Gaeltacht areas.
Gaelcholáistí	Gaeilge-medium second level schools located outside of the Gaeltacht areas.
Gaeilgeoirí	Students who learn through the medium of Gaeilge at primary and/or second level education.
Immersion Education	Students opt to learn through the medium of a second language with the aim of developing bilingualism.
Maintenance Heritage Language	Type of bilingual education system in which native speakers of a minority language receive education through the medium of that language in their community. For example Gaeilgeoirí from a Gaeltacht area attend a primary and secondary school in that Gaeltacht.
Mathematics Register	The language and vocabulary specific to mathematics including words, phrases and methods of arguing within a given situation, conveyed through the use of natural language e.g. Gaeilge or English.
Monolingualism	Having only one language, English in this study.

Subtractive Bilingualism	Occurs when an individual's first language and culture are replaced by the new language and culture, usually occurring in a pressurised context. Negative self-concept may develop.
Submersion Education	Schools/Institutions that contain bilingual students of a minority language, who are required to learn through the majority language.

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1.

Introducing Mathematics Education and Bilingualism

1.1 Introduction

For generations raised on 'Peig*', a significant language transformation has taken place in Ireland – Gaeilge (Irish) has become trendy. Normality is engraved in sending our children to Gaeilge-medium schools, seeing comedians doing gigs 'as Gaeilge', and tuning into television programmes presented by fluent Gaeilge speakers. In a very short period we have progressed from shunning our native language to endorsing it as a fashionable and positive thing for our country. Where did it all go right for Gaeilge? It is difficult to pinpoint exactly where it all started but what is clear is that this impromptu revolution came about through a combination of significant socio-political developments. A major catalyst has been the explosion of Gaeilge-medium schools which has dramatically changed the face of our primary and second level education system. Previously Gaeilge-medium education was mainly limited to remote isolated parts of Ireland known as Gaeltachtaí (all-Irish speaking districts) and these institutions were viewed by outsiders as strange and archaic. However, sending your children to Gaeilge-medium education (outside of these Gaeltacht areas) is now as necessary as possessing the latest iPod or mobile phone.

This research monograph will present findings from a doctoral study undertaken on bilingualism and mathematics education in the Irish context (see Ní Ríordáin, 2008). MacNamara (1966) undertook the only study carried out in Ireland in relation to teaching and learning mathematics in a second language. However, the context of his study is not replicated in the current study undertaken by the author. MacNamara's

* Core textbook for the Gaeilge syllabus at second level – extremely difficult and hated by many who studied it.

study was conducted with students whose mother-tongue was English and who were learning mathematics (and all other subjects) through the medium of Gaeilge (not necessarily by choice). The overall aim of his study was:

"to discover the effect on arithmetical attainment of teaching arithmetic through the medium of Irish to children from English speaking homes" and *"to discover the effect of the entire programme for reviving Irish in national schools on the level of English attainment."*

(MacNamara, 1966, p.6)

His testing involved the use of mechanical and problem arithmetic. He concluded that the above students were behind on problem arithmetic by about eleven months, but not in mechanical arithmetic. He concluded that overall compulsory teaching through the medium of Gaeilge was detrimental to student learning. A number of studies have been undertaken in relation to reading achievement in Gaeilge and English (e.g. CILAR, 1975; Cummins, 1977b; 1978; C.E.B., 1985), and/or the position of Gaeilge in schools, but none have ventured towards assessing attainment in other subject areas dependent upon the medium of instruction and language competence (Ní Mhurchú, 2001).

The author's doctoral study was focused on Irish post-primary and undergraduate bilingual mathematics students' (Gaeilgeoirí) experiences of learning mathematics through the medium of English, their second language of learning (Ní Ríordáin, 2008). This includes examining the relationship between language proficiency and performance on mathematics word problems; investigating additive and subtractive bilingualism in an Irish context; examining sources of difficulty encountered with the English language mathematics register; and in the process developing significant insights into cultural and pedagogical influences on the transition from Gaeilge-medium to English-medium mathematics education. The bilingual students in this study initially studied mathematics through the medium of Gaeilge (Irish) either at primary level only, or at primary and second level Gaeilge-medium education. The research undertaken by the author provides an account of research on bilingualism and mathematics education in Ireland and is designed to build on research undertaken in other cultural contexts, while suggesting some productive lines for

further enquiry. Mathematics is culturally dependent and specific to the environment in which it is taking place. Thus it is necessary for each country to undertake relevant research in relation to bilingualism and mathematics education appropriate to the educational context in operation. Such research makes a significant contribution to the domain of mathematics education at a national and international level.

1.2 Background to the Irish Context

"The Irish language as the national language is the first official language." (Bunreacht na hÉireann, Article 8, 1937). Since the foundation of the Irish Free State in 1921, the education system has been recognized and utilized as a basis of the movement for fostering Gaeilge-English bilingualism (Education Act, 1998, Pr. 1, Section 6). Education has always been highly valued in Ireland. A three tiered education system has been established where primary education lasts for eight years for children between the ages of four and twelve. The second level school span is predominantly a six-year cycle, taken by children aged twelve to eighteen. Third level education is provided mainly by universities, institutes of technology and colleges of education. A characteristic feature of the Irish primary and post-primary school systems is that the curriculum can be mediated in either Gaeilge or in English. However, two diverse contexts exist for Gaeilge-medium education in Ireland - namely Gaeltacht schools and Immersion Education (Gaelscoileanna/Gaelcholáistí).

Teaching and learning through the medium of Gaeilge is the natural school environment for teachers and pupils in the Gaeltacht areas (Irish speaking districts) of Ireland. Gaeilge is the mother-tongue spoken in the home, in the workplace and in the community and, therefore, it is natural that children learn through this medium. This heritage language is held in high regard both by the members of the communities and by the teachers in Gaeltacht schools. The Gaeltacht areas are revered as a primary agency for maintaining the Gaeilge language in Irish society.

Fig.1.1 Location of Gaeltacht regions in the Republic of Ireland

A parents' initiated movement in the early seventies was responsible for the establishment of Gaeilge-medium primary and secondary schools *outside* of the Gaeltacht regions, known respectively as Gaelscoileanna and Gaelcholáistí (Coady & Ó Laoire, 2002). The number of students attending these schools has seen an increase of more than sixty percent in the last decade, with the number of schools increasing by more than fifty percent (Fás ar an nGaelscolaíocht sa Ghalltacht, 2006). Students attending these schools are predominantly from English speaking households and the schools are located in English speaking communities. Parents of pupils attending these schools view Gaeilge as an important language and the primary aim in enrolling their children in immersion education is to develop bilingualism. Clearly the general public's interest in their native language is still strong, as is their desire for their children to learn through the medium of Gaeilge.

However, learning issues may arise both for students and teachers at the transition from Gaeilge-medium education to English-medium education. This is inclusive of both key transition periods – primary level to second level education, and second level to third level/further education. Students have the option of transferring to learning through the medium of English at second level, whereas English-medium education is the norm at third level. The author anticipated significant difficulties arising in the Science, Engineering and Technology (SET) subjects – mathematics in particular. The SET subjects employ specific language registers - for example there

exists a chemistry register, a biology register, and a mathematics register in Gaeilge and in English. Difference in the registers may occur between the languages and students are required to be literate in these registers in Gaeilge and/or in English. Thus Gaeilgeoirí submerged in the transitions under investigation will be required not only to learn mathematics, but also to learn mathematics through the medium of English (Barwell, 2003).

1.3 A Short Note on the History of Gaeilge as a Medium of Learning

In order to understand the development of Gaeilge as a medium for teaching and learning mathematics in Ireland it is first necessary to give a brief political history of the Irish language in Ireland. For the purpose of this monograph and to draw on the research undertaken by the author, she will specifically address the context in the Republic of Ireland, while illuminating comparisons with Northern Ireland where appropriate. Perhaps this is the most apt place to draw attention to the political role that successive governments have played in the development of Gaeilge and various language policies, but it is crucial to examine the actions leading up to the political divide of the country in 1921. Up until the 16th century, Gaeilge and its associated culture and traditions were dominant throughout the island of Ireland, surviving invasions by Viking and Norman groups. However, English colonisation of Ireland began in the mid 16th century and continued into the 17th and 18th centuries by driving the Irish from their lands and replacing them with English and Scottish colonists. The persecution of the Irish people was relentless and coupled with the Great Famine that swept the country during 1845 to 1852, a dramatic decline in the number of Gaeilge speakers and use of the Gaeilge language in Ireland was observed.

A number of significant Gaeilge language organisations were established during this period in order to halt the decline of the use of the language including the Ulster Gaelic Society (1830) and Conradh na Gaeilge (The Gaelic League, 1893) who published documents in Gaeilge and promoted its use in everyday and academic settings. Since the foundation of An Saor Stáit (The Free State) in 1921, a divide has been established in Ireland – the Republic of Ireland (26 counties) and Northern

Ireland (6 counties). Gaeilge is the first official language (English the second) of the Republic of Ireland and with the establishment of An Saor Stáit (1921) it was intended to restore the Gaeilge language and its use throughout the country. This ambitious aim was never achieved and currently Gaeilge is spoken natively by a small but increasing minority (95,503 people) of the population and specifically in 7 regions (official Gaeilge speaking districts in the Republic of Ireland) known as Gaeltachtaí (CSO, 2006). A number of positive social and political developments in relation to Gaeilge have taken place in the past 10 years including consolidation of the language at constitutional level (Official Languages Act, 2003); legal contexts (appointment of an Official Languages Commissioner); and at European level with the establishment of Gaeilge as an Official Language (2006) of the European Union (Harris, 2007). Other significant revitalisation movements have been largely targeted through education (which will be discussed in the next section); through the media - mainly TG4 (national television station) and Radió na Gaeltachta (national radio station); and an increased use of Gaeilge in the public sector through bilingual (Gaeilge and English) provision of advertising and services. All of these are positive developments for the language and have contributed to its increased use nationally and internationally.

The pressure of years of foreign occupation, along with the complicated political, religious and economic pressures of the 18th and 19th centuries, had rendered Gaeilge non-existent at the top of the social scale in Ireland. More importantly, it had weakened its position among the entire population (Ó Cuiv, 1969). The State (under British Rule) became involved in the provision of education for the first time in the early nineteenth century, which led to the setting up of the National School system (1831). This education system is described as having "a British cultural emphasis" and having "crushed the Irish language" (Kelly, 2002, p.4). The introduction and use of the "Bata Scóir" (a tally stick used to hit students depending on the number of times they spoke Gaeilge) by teachers quickly spread as primary schools were set up throughout the country, resulting in the prohibition of Gaeilge as medium of instruction and communication. Parents supported this punishment system, as Gaeilge was associated with poverty and English increasingly with economic prosperity. Secondary education, unlike primary education, at that time was reserved mainly for the rich and those who could afford to pay to attend second level

education. Secondary schools were not widespread and therefore only a select minority continued with second level education. However, like the National Schools, Gaeilge was banned from being taught and spoken within the schools and emphasis was placed on the English language. Therefore, during the nineteenth century it was evident that English was rapidly replacing Gaeilge as the native language, and the strict prohibition of Gaeilge in the education system was perceived as being instrumental in this change (Kelly, 2002).

When An Soar Stáit was established in 1921, Gaeilge was recognised as the first official language, with the intention of restoring it throughout the country (Purdon, 1999). The new state adopted a programme for restoring Gaeilge that was aimed almost exclusively at school children. The plan was to immerse all children in Gaeilge for the entire period of their schooling, so that in the space of a generation or so, the language would be brought back to everyday use (MacAogáin, 1990). The schools and education were chosen to revive the language as it was felt that they had been responsible for displacing Gaeilge with English. Also it was believed that teaching Gaeilge as a subject alone was not sufficient for reviving the language so more comprehensive measures would be needed and therefore all subjects, including mathematics, were to be taught through this language medium (Kelly, 2002). The debate on using Gaeilge as a medium of instruction in primary schools and a lack of implementation in all schools, continued through the subsequent decades. However, in November 1959, Dr. Patrick Hillary, the then Minister for Education, proposed that schools and teachers should concentrate on teaching Gaeilge well rather than teaching through the medium of Gaeilge (Kelly, 2002). Subsequently, two months later the Government abolished the use of Gaeilge as a medium of instruction in all but a minority of primary schools (Gaeltacht schools). Overall it was felt that Gaeilge, as a medium of instruction, had reduced the standard of education with little improvement in the use and status of the language outside of education (MacNamara, 1966).

At the time of the establishment of the Free State the emphasis on Gaeilge in second level schools was less intense than it was in primary schools. However, from 1927 Gaeilge became a compulsory subject for the award of the key state examination

certificates (Department of Education, 1975). From 1924 the Government provided additional grants to schools using Gaeilge, with the amount received dependent on the level of Gaeilge being used and spoken. As a result the number of students sitting Gaeilge in examinations increased by 15% within the first ten years of Independence (Kelly, 2002). This trend continued through the 1930s and 1940s with the number of schools teaching through the medium of Gaeilge and the number of pupils sitting the Gaeilge examination increasing steadily. Clearly and negatively what was sustaining this were the financial rewards that the Government offered those willing to use Gaeilge to the greatest extent possible within the schools. However, Gaeilge-medium education at second level was primarily limited to the Gaeltacht regions of Ireland. Since the 1920s secondary schools (and primary schools) were obliged to teach Gaeilge. Compulsion was the most "consistent trait" of any of the language policies introduced (Kelly, 2002, p.14). Gaeilge was a compulsory curriculum subject, a compulsory examination subject and a requirement in order to receive certification. It wasn't until 1973, when Richard Burke was the Minister for Education, that the requirement to pass Gaeilge in order to pass the Leaving, Intermediate and Group Certificate examinations at second level was dropped. However, an honour in Higher Level Gaeilge is still required to enter primary level teacher-training colleges. So the element of compulsion is still present for many students.

A significant development in relation to Gaeilge and Gaeilge in schools is the increase in the number of Gaelscoileanna (primary schools teaching through the medium of Gaeilge) and Gaelcholáistí (second level schools teaching through the medium of Gaeilge) – Gaeilge-medium immersion education outside of the Gaeltacht regions. In 1972 there were 11 primary and 5 secondary schools providing education through Gaeilge outside of Gaeltacht areas. However, the rise in popularity of immersion education is significant and has seen an increase in excess of 60% in student numbers over the past decade. Currently, 136 Gaelscoileanna and 50 Gaelcholáistí (13 functioning as units within English-medium schools) have been established in the Republic of Ireland with an estimated 33,000 pupils attending these schools (Gaelscoileanna Teo., 2008, see Fig. 1.2.).

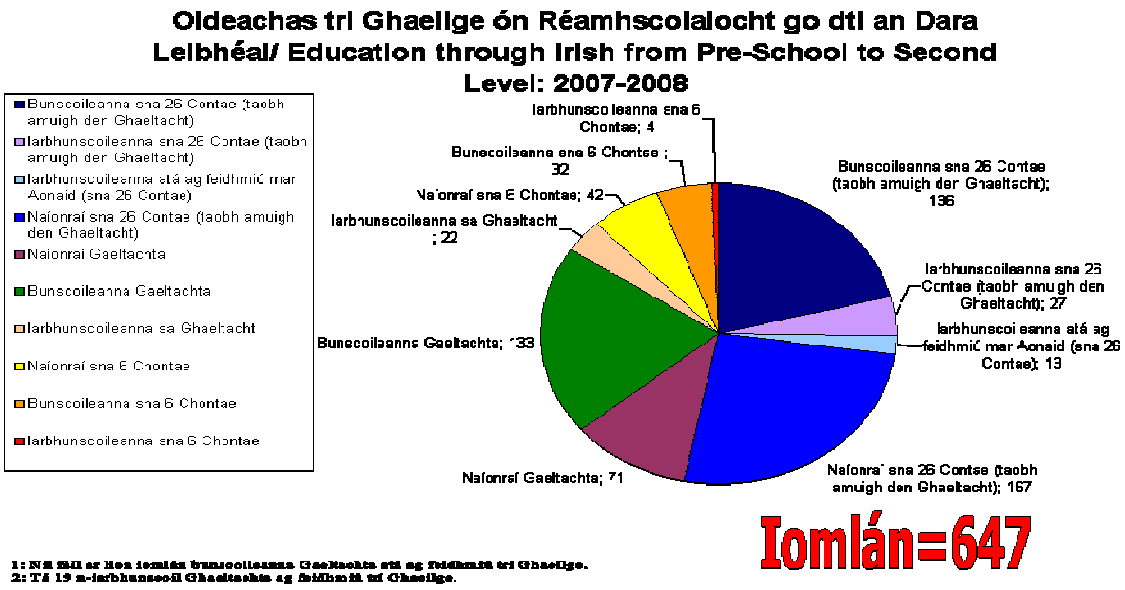


Fig 1.2 Number of Gaeilge-medium pre-schools (Naíonraí), primary (Bunscoileanna) and second level (Iarbhunscoileanna) schools in the Republic (26 contae) and Northern Ireland (6 contae).

Combining this with the number of students attending primary and second level schools in the Gaeltacht areas, approximately 7% of the total primary level population and 2.5% of the second level population are learning mathematics through the medium of Gaeilge. Also, in tandem with this is the development of Naíonraí Gaelacha (Gaeilge-medium play schools) for pre-schoolers. Immersion Gaeilge-medium education is largely a parent initiated voluntary movement provoked by the lack of success of State language policies since 1922 (Ní Mhurchú, 2001). This suggests that the general public's interest in the native language is still strong, as is their desire for their children to learn through the medium of Gaeilge.

Since the 1970s Gaeilge has been taught as a school subject only (Gaeltacht schools and Gaelscoileanna/Gaelcholáistí being the exceptions). It is part of the core curriculum during the years of compulsory schooling, six to sixteen. Even though Gaeilge as a medium of instruction in the Irish school system has undergone many changes, significant numbers of students are learning mathematics through this medium. However, what is clear and of importance is that the Irish Government has

played a significant role in establishing Gaeilge as a medium of instruction in primary and secondary schools. Clearly,

"Decisions about which language to use, how, and for what purpose(s), are political. This political role of language is not dealt with in the literature on bi/multilingualism and the teaching and learning of mathematics."

(Setati, 2002, p.13).

1.4 Socio-Political Concerns

The decisions about which language(s) are used in education are predominantly political in nature (Edwards, 1994). As demonstrated in the previous sections the history of the Gaeilge language in Ireland and as a medium for learning mathematics has been marred by issues of access, power and dominance. In the 19th and 20th centuries, English was the dominant language of learning within the country, regardless of mother-tongue spoken. With the establishment of the new state (1921) Gaeilge-medium education was central to policy plans for the revitalisation of the language and compulsion was the norm throughout the country but detrimental to student learning (Kelly, 2002; MacNamara, 1966). Therefore the last two hundred years has borne witness to the Gaeilge language experiencing two extremes – it being against the law *to* teach/speak Gaeilge and it being against the law *not to* teach/speak Gaeilge. Both measures were introduced by the then governments and were implemented through the use of education and the schools. It is evident that the governments played a crucial role in the position of Gaeilge in schools and as a consequence its status in society. Removal of this element of compulsion in the 1960's was a catalyst for change. At this time access to Gaeilge-medium education was primarily reserved for people living in the Gaeltacht regions of Ireland due to social and cultural necessity, but was looked upon by many outside these regions as backward and restrictive given the English language association with universality and economic prosperity (Kelly, 2002). However, removing compulsion signalled an element of eliminating 'choice' for parents outside of Gaeltacht regions and thus fuelling the need for Gaeilge-medium immersion education.

Clearly, some of the Irish government interventions were not always done with sufficient tact or wisdom that might have made them more effective. Take the example of the position of Gaeilge in the schools of six counties in the North. Irish

was tolerated as an optional foreign language only and as an acceptable subject in secondary schools. By the 1950s, Gaeilge was as popular and chosen as often as French was. And as Lord Charlemont, the Stormont Minister of Education said, "forbidding it (Irish) under pressure will stimulate it to such an extreme that the very dogs – at any rate, the Falls Road dogs – will bark in Irish" (as cited by Purdon, 1999, p.59). The first Gaeilge-medium primary school in Northern Ireland was established in 1971 which saw an intake of only 9 pupils (Ó Baoill, 2007). But the growth and recognition of Gaeilge-medium immersion education in the North has been as phenomenal as in the Republic, and has lead to the development of a small but unique urban community of Gaeilge speakers in Northern Ireland. Similarly in the Republic of Ireland, immersion Gaeilge-medium education was stimulated when the compulsion element was removed, and accordingly the students' (outside of Gaeltacht regions) option of learning through the medium of Gaeilge. Social structures emphasise the importance of choice and access to Gaeilge-medium education and that children should not be denied this opportunity for learning. Hence, Gaelscoileanna emerged in the 1970's and were independent of other primary schools in their locality. Initially, students enrolled in immersion education were restricted to those coming from Gaeilge speaking homes and had a strong grasp of the Gaeilge language (Ó Baoill, 2007). Therefore, discrimination was evident in the early days but policies changed due to demand for access and the general publics' interest in sending their children to immersion education and developing bilingualism (Gaeilge and English).

Although the initial establishment of Gaeilge-medium education (outside of Gaeltacht regions) arose out of social influences (largely community and parental initiatives) and a resistance to political policies, growth and development have been stimulated by financial and facilitative support by various Governments and governmental bodies throughout the late nineties and early twenty first century (Ó Baoill, 2007). Another consideration is that mathematics learning and teaching are socially and culturally situated, and mathematics cannot be considered culture free (Bishop, 1988). Barton (2008) sums it up well - "The practical reality is that every indigenous peoples' context is different" (p. 167). The range of difference is broad. For example differences may exist in relation to language use; differences in relation the political situation in the context; differences in education; and many more

(Barton, 2008). Children growing up in the Gaeltacht areas of Ireland are immersed in a different language and culture to those growing up in all-English communities. Similarly, children attending the Immersion schools will have different experience to those in Gaeltacht areas and all-English areas. Accordingly, it is anticipated that these two different Gaeilge groups within the Irish context will possess a different world-view, and accordingly a different mathematical world, to those from an all-English environment within the country.

1.5 Research Purpose

The work presented in this research report was originally motivated by the author's reflections on her own experience of coping with a new language of learning in mathematics. Initially the investigation focused on the effect of changing the language of instruction on mathematics learning (Phase 1) and further exploratory research focused on identifying the specific difficulties encountered by Gaeilgeoirí (students who learn through the medium of Gaeilge) when transferring to English-medium education (Phase 1). This led to more in depth investigations at the primary to second level and second to third level interfaces so as to get a clear understanding of the experiences and challenges faced by Gaeilgeoirí (Phase 2).

The research project was an evolving investigation shaped by previous findings and other considerations. The research aims of the preliminary investigation (undergraduate dissertation) included:

- To investigate the extent to which performance in mathematics could be attributed to the language of instruction, and
- To examine the experiences of Gaeilgeoirí in the transition process at third level.

Follow-up exploratory research (Maths Life Histories) addressed the following research aims:

- To establish and clarify the key issues facing Gaeilgeoirí in the transition from learning mathematics through Gaeilge to learning mathematics through English.

- To ensure that subsequent research to be carried out by the author addresses the relevant issues and contributes to development in the research domain.

The final phase of the research and data collection (Phase 2) was guided by the following **key research questions**:

1. Is performance on mathematics word problems for Gaeilgeoirí through the medium of English affected by their level of language proficiency in English and in Gaeilge? If so, what is the effect and degree of influence that bilingualism has on mathematics learning for Gaeilgeoirí in this study?
2. Is there a significant difference in mathematical learning through the medium of English between Gaeilgeoirí transferring from Gaeltacht schools (subtractive bilingualism) and Gaeilgeoirí transferring from Gaeilge-medium schools (additive bilingualism)?
3. Do particular features of the English mathematics register cause difficulty for Gaeilgeoirí at first year second level education and first year undergraduate education?
4. Do cultural and pedagogical factors influence Gaeilgeoirí's transfer from Gaeilge-medium to English-medium mathematics education?

1.6 Significance of the Research

The number of students enrolled in Gaeltacht primary and second level schools has remained consistent over the past decade. Currently there are 133 primary schools and 22 secondary schools located in Gaeltacht areas (Mac Donnacha *et al.*, 2005; Gaelscoileanna Teo., 2008, see Fig. 1.2). However, the rise in popularity of immersion education is significant with 168 Gaelscoileanna (primary schools) and 44 Gaelcholáistí (second level schools) now established throughout Ireland (Gaelscoileanna Teo., 2008, see Fig. 1.2). By combining both the number of Gaeltacht students with immersion students one unveils a significant and increasing minority of our primary and second level schools' populations learning through the medium of Gaeilge; approximately forty eight thousand students in total (MacDonncha *et al.*, 2005). This equates to 7 percent of the total primary level population and 2.5 percent of the second level population.

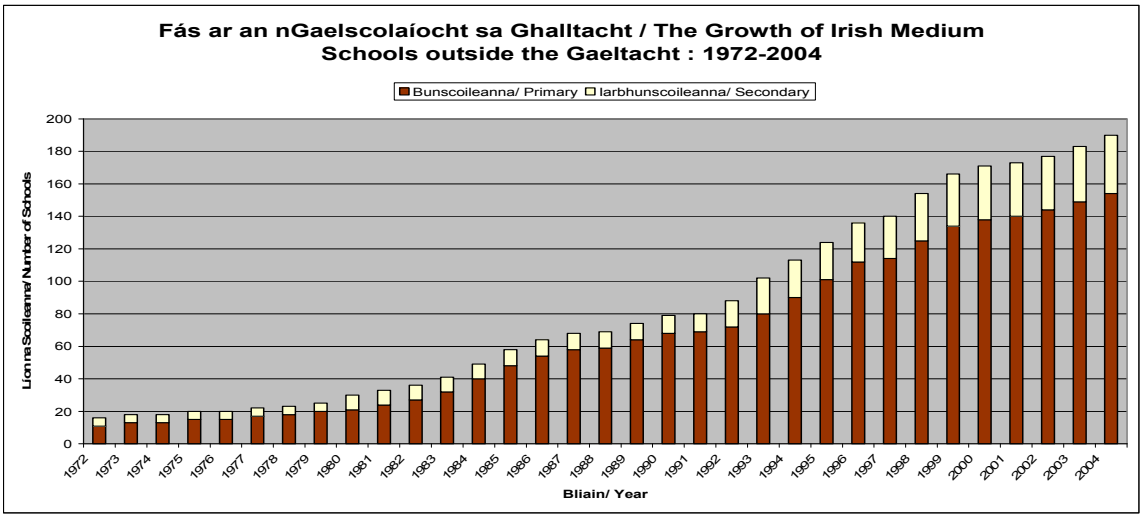


Figure 1.3 The growth of Gaeilge-medium schools outside of Gaeltacht areas (Available at www.gaelscoileanna.ie).

The above graph demonstrates the substantial growth in Gaeilge-medium education. As is evident annual increases in pupil numbers have been recorded and this trend is expected to continue in the future. What is of importance to the author is that the majority of Gaeilgeoirí will face an impending transition to English-medium education, either at second or third level. Current education statistics reveal 6.9 percent of final year primary level students are learning through the medium of Gaeilge, while at second level 1.5 percent of students sit their final examinations through the medium of Gaeilge (Tuarascáil Staitistiúil, 2007/2008). This suggests that there may be significant numbers of Gaeilgeoirí transferring to English-medium education annually at both transition points. However, given that this is the first study of its type to be undertaken in Ireland and that no definite statistics exist on the number of students transferring, it highlights the need for a national investigation into the school types attended by Gaeilgeoirí.

This study is relevant to those involved in mathematics education at all levels, in particular those working with students learning through the medium of a second language. The study delivers insights and recommendations that may be useful for further research to be carried out in this area of education in Ireland. This research project is of importance to those who, like the author, want to see a fairer education system introduced that caters for the needs of Gaeilgeoirí in the transition from

Gaeilge to English-medium education, so as to enhance their learning and understanding. This issue of educational equity needs to be addressed and answered, and this is not just a local issue as it resonates with equity issues worldwide.

When considering the importance of mathematical literacy (OECD, 2006), it is difficult to separate the function of linguistic and cultural factors in the development of mathematical thinking, learning and understanding. If Gaeilgeoirí fail to develop mathematical literacy then they may be restricted in how they can participate in the knowledge society. This is the first study of its type to be undertaken in Ireland in relation to bilingualism and mathematics education. The author addressed the mathematical needs of Gaeilgeoirí at a local level in order to identify the challenges they are confronted with in the transition to English-medium mathematics education. This is to ensure the development of support measures and pedagogic resources in order to enhance Gaeilgeoirís' mathematical competence and preparation to engage in the knowledge society and economy. Although a large body of research has been carried out internationally and recommendations drawn up, it is important that each country undertakes their own research at a local level in order to address the specific needs of the mathematics learners within that context (Ellerton & Clarkson, 1996). The research undertaken by the author replicates international studies and therefore contributes to the development and strengthening of international theories and findings, in particular in the area of cognition and bilingualism. This investigation provides an Irish perspective on bilingual issues prevalent in international mathematics education research.

1.7 Limitations of the Study

There are a number of limitations that readers need to be aware of when discussing the findings of this research monograph. These include:

- Different sample sizes were utilized when investigating both transitions and although some similar findings are emerging from both transitions, the samples are too small to draw broad generalisations.
- The studies were conducted in a number of different second level and third level institutions around the country at a certain time (these students had just

transferred to English-medium mathematics education; therefore changes may occur over time) and thus may not be representative of all Gaeilgeoirí in the transition to English-medium second and third level education.

- The tests administered at each transition were not inclusive of *all* mathematics topics and language skills e.g. language skills used for academic listening but appropriate tests were designed, piloted and redesigned at each transition.

Although the sample of students involved in this study is relatively small from which to draw generalizable conclusions about all Gaeilgeoirí in the transition to English-medium mathematics education at second and third level, the author considers the findings reported present a good description of bilingualism and its influence on mathematics learning in Ireland, and a credible basis for decision-making.

1.8 Overview of the Report

This Chapter has presented the background to the study and discussed its significance and the research questions addressed. An outline of the other Chapters in the monograph follows.

Chapter 2 explores the concept of mathematics as a language and the significance of recognising a mathematics register, and its influence on teaching and learning, particularly in a second language. Psycholinguistic theories and cultural influences are also examined. This chapter also includes a review of current literature on the issue of teaching and learning mathematics in a second language.

Chapter 3 deals with issues concerned with bilingual education and issues surrounding second language acquisition and cognitive theories of bilingualism.

Chapter 4 gives a detailed description of the methodology and research design employed in this investigation. A mixed-methods approach was utilized in order to give an in depth account of the situation that exists in Ireland.

Chapter 5 presents the findings related to Phase 1 and 2 of this investigation. Data was gathered at both transitions and their relevance to international findings is highlighted as appropriate. The findings discussed in this Chapter give a significant insight into the issue of bilingualism and its influence on mathematics learning within the Irish context.

Chapter 6 presents the conclusions and recommendations for future research as concluded from the findings of this investigation.

A comprehensive bibliography in this area of research has been assembled and is included at the end of the document along with the appropriate appendices.

2.

Mathematics and Language

2.1 Introduction

Language and communication are essential elements of teaching and learning mathematics, and this is evident from research carried out in bi/multilingual settings (Gorgorió & Planas, 2001). Language is employed as a communication tool and facilitates the transmission of (mathematical) knowledge, values and beliefs, as well as cultural practices. Language is also the channel of communication within a mathematics classroom as language provides the tool for teacher-student interaction (Smith & Ennis, 1961). Competence in the language of communication/interaction is a prerequisite for engagement in the learning process. Both the Gaeilge and English languages will play a significant role in the transition from Gaeilge-medium to English-medium mathematics education. For mathematical learners this is twofold in that they are required to have competence in the language of instruction and in the language of mathematics (specifically the mathematics register). The development of mathematical learning and understanding is therefore interrelated with language capability. Through the examination of psycholinguistics theory, the role of language in relation to thinking and understanding is exposed. This chapter presents the specific background knowledge and theory underpinning the study undertaken by the author.

2.2 Mathematics as a Language

Language is defined as the "formal organisation of symbols, sounds and gestures used to communicate ideas, thoughts and feelings, to create meaning" (Ellerton & Wallace, 2004, p.1). Thus mathematics appears to be a type of formal language as it too consists of symbols, sounds and gestures that are used to communicate and devise mathematical concepts. Sternberg (2003, p.286) identifies six properties that are distinctive of language:

1. *Communicative:* language permits us to communicate with one or more people who share our language.
2. *Arbitrarily symbolic:* language creates an arbitrary relationship between a symbol and its referent: an idea, a thing, a process, a relationship, or a description.
3. *Regularly structured:* language has a structure; only particularly patterned arrangements of symbols have meaning, and different arrangements yield different meanings.
4. *Structured at multiple levels:* the structure of language can be analyzed at more than one level (e.g. in sound, in meaning units, in words, in phrases).
5. *Generative, productive:* within the limits of a linguistic structure, language users can produce novel utterances, and the possibilities for creating new utterances are virtually limitless.
6. *Dynamic:* languages constantly evolve.

The above properties are applicable to mathematics and to the concept of mathematics as a language.

But is mathematics truly a language? Perhaps because mathematics is associated with written work, involves symbols as opposed to words and is communicated on paper rather than orally, it is difficult to perceive it as a language. Pimm (1987, p.75) argues that it is not since "there is no one group of people for whom mathematics is their first language". Instead we consider mathematical language as a distinct 'register' within a natural language e.g. Gaeilge or English, which is described as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings." (Halliday, 1975, p.65).

2.3 The Mathematics Register

One aspect of the mathematics register consists of the special vocabulary used in mathematics (Gibbs & Orton, 1994) and it is the language specific to a particular situation type (Lemke, 1989). But it is more than just vocabulary and technical terms. It also contains words, phrases and methods of arguing within a given situation, conveyed through the use of natural language (Pimm, 1987). The grammar and

vocabulary of the specialist language are not a matter of style but rather methods for expressing very diverse things (Ellerton & Wallace, 2004). *Each* language will have its own distinct mathematics register and ways in which mathematical meaning is expressed in that language. Taking the English language mathematics register it includes:

> *"the use of common words with specialised meanings; syntax characteristics include increased use of logical connectives, while discourse characteristics include increased density of meaning, increased use of passive voice, and the need for multi-directional reading."*
>
> (Barton & Neville-Barton, 2003, p. 4)

Within the mathematics register different forms of mathematical language can be found (Figure 2.1.).

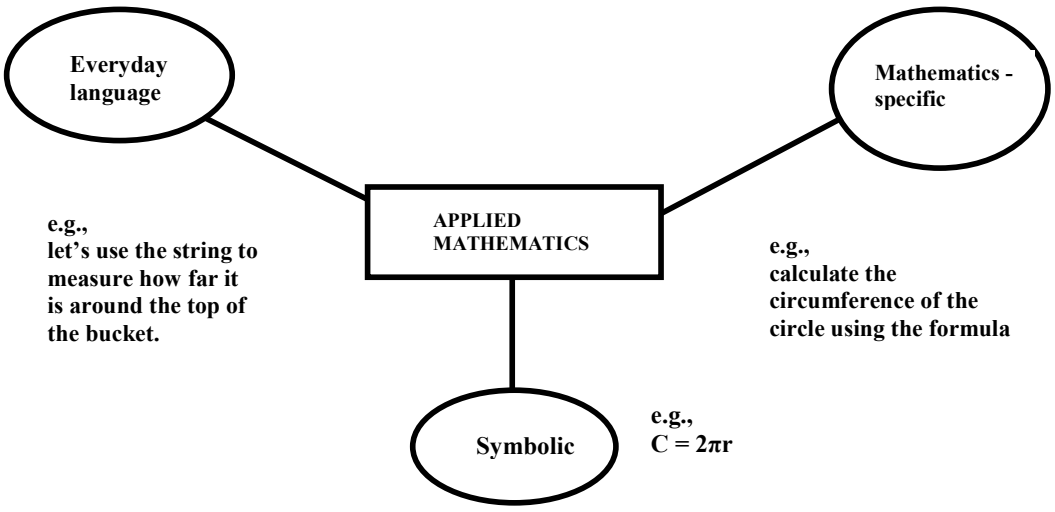


Figure 2.1 Diagram showing the types of mathematical language (Meaney, 2005 (adapted from Bubb, 1994))

As is evident the complex 'register' of mathematics is similar to a language and requires skills of learning similar to those used in learning a language. This adds another dimension to mathematics learning, and reinforces the view that the content of mathematics is not taught without language. The process of learning mathematics

involves the mastery of the mathematics register (Setati, 2005a). This allows students to communicate their mathematical findings in a suitable manner but "without this fluency, students are restricted in the ways that they can develop or redefine their mathematical understandings." (Meaney, 2005, p.129). By developing a child's mathematical register it provides them with analytical, descriptive and problem solving skills within a language and a structure so that they can explain a wide range of experiences. Once the register is mastered, learners will have the ability to listen, question and discuss, together with an ability to read and record.

Similarly, classroom discourse is an intricate structure. It consists of units of language such as those used in conversations, lectures, stories, essays and textbooks (Sternberg, 2003). Grammatical sentences are structured according to systematic syntactical guidelines – likewise episodes of discourse are systematically structured. The context of learning is multidimensional. Understanding discourse does not rest solely on the interpretation of words written in textbooks and spoken by the teacher but also on the knowledge of the physical, social or cultural context within which the discourse takes place (Sternberg, 2003). For example, the functions assigned to learners and teachers in the given environment, modes of communication, intentions, linguistic choices, and contexts of communication/learning are all influential in the interpretation of meaning (Georgakopoulou & Goutsos, 1997). By taking into consideration who is producing the mathematical text (spoken or written), the context that it is being produced in and the medium through which it is expressed we may be able to evaluate the relationship between students' mathematical understanding and its relationship with language. However, mathematics learning/understanding and its relationship with language is an extremely complex and diverse area of research and it will be examined in the subsequent sections.

2.3.1 Language Features that impede Mathematical Learning

Mathematics is not "language free" and due to its particular vocabulary, syntax and discourse it can cause problems for students learning it in a second language (Barton & Neville-Barton, 2003). While many students who learn mathematics in their mother-tongue (e.g. Gaeilge) have difficulty in acquiring the mathematics register, this is heightened for those who must learn it in a second language (e.g. English). Learners have to cope with the new mathematics register, as well as the new

language in which the mathematics is being taught (Setati & Adler, 2000). Some of the language features that may impede mathematical learning are discussed in the following paragraphs.

Borrowed Words/ Ambiguous Terms

A key issue that causes significant problems for second language learners (as well as monolingual learners) is the number of 'borrowed' words from everyday English (Pimm, 1987). These words tend to be ambiguous due to having one meaning in the mathematics register, while another meaning in its everyday use (Yushau & Bokhari, 2005). Table 2.1 is a compilation of the more common ambiguous words found in mathematics education. The non-mathematical meanings of these terms can influence mathematical understanding, as well as being a source of confusion.

Table 2.1 Some ambiguous words used commonly in school mathematics (Durkin & Shire, 1991, p. 74).

above, altogether, angle, as great as, average, base, below, between, big, bottom, change, circular, collection, common, complete, coordinates, degree, difference, different, differentiation, divide, down, element, even, expand, face, figure, form, grid, high, improper, integration, leaves, left, little, low, make, match, mean, model, moment, natural, odd, one, operation, overall, parallel, path, place, point, power, product, proper, property, radical, rational, real, record, reflection, relation, remainder, right, root, row, same, sign, significance, similar, small, square, table, tangent, times, top, union, unit, up, value, volume, vulgar

Also, Rudner (1978) found that:

- Conditions (if, when);
- Comparatives (greater than, the most);
- Negatives (not, without);
- Inferentials (should, could, because, since);
- Low information pronouns (it, something); and
- Lengthy passages

are sources of difficulty and hinder students' interpretation and understanding of mathematical word problems.

Specialist Terms

The use of specialist terms can lead to misunderstanding and misinterpretation of mathematical tasks. Students tend to only encounter these terms within the mathematics classroom (for example, "quadrilateral", "parallelogram" and "hypotenuse") and they are unlikely to be reinforced outside of it (Pimm, 1987). If second language learners do not acquire their correct meaning then this can lead to difficulties within the mathematics context. Second language learners have a tendency to translate new mathematical terms/vocabulary into their mother-tongue. This translation may not exist and/or it may be done incorrectly, thus resulting in further confusion and misinterpretation (Graham, 1988).

Context

Context is also a key issue. 'Words can change their meaning depending on their context within the mathematics lesson' (Gibbs & Orton, 1994, p.98). In terms of language analysis, this is known as semantics – establishing the meaning in language, or the relationship and representation between signs and symbols. Due to the multiple meanings that various words can have, the context is vital in determining the correct interpretation. Findings from a review of literature found that children experience more difficulties with the semantic structure of word problems than with other contributing factors such as the vocabulary and symbolism of mathematics and standard arithmetic (Ellerton & Clarkson, 1996).

Symbolism

Symbolism is one of the most distinctive features of mathematics. It is crucial for the construction and development of mathematics. Unfortunately "symbolism can accordingly cause considerable difficulties to those whose mother language has different structures" (Austin & Howson, 1979, p.176). One of the requirements for mathematical learning is that students can interpret the mathematical text and convert it to an appropriate symbolic representation, and perform mathematical operations with these symbols (Brodie, 1989). Thus if students cannot understand the text (due to the language medium) they will be unable to convert it to the appropriate mathematical construction needed to solve the problem. Symbols provide structure, allow manipulation, and provide for reflection on the task completed.

Registers exist in many disciplines (e.g. science, technology, etc.) but likewise ordinary/everyday English language can be classified as a register. The mathematics and ordinary language registers can interfere, often in subtle ways, in a learning environment. Thus learners need to recognise each of these registers so as to identify which is being used at any given time (Sierpinska, 1994), and this a challenge many Gaeilgeoirí encounter when transferring to learning mathematics through the medium of English. But these are not the only factors that influence mathematical learning. The following sections present a number of key theories that need to be taken into consideration when addressing the relationship between language and mathematics.

2.4 Psycholinguistic Theories

Psycholinguistic theories and the influence of language on mathematical thinking and understanding are examined in the following sections. These theories provide us with the tools to examine how bilingualism may affect mathematical learning for Gaeilgeoirí.

2.4.1 Language and Thought/Thinking

Influential psychologists and educationalists, including Vygotsky and Bruner, have investigated the nature and relationship between language and thought. The primary concern to emerge from this research is whether language follows thought, thus making language a means for expressing our thoughts, or whether language determines and is a prerequisite for our thoughts (Brodie, 1989). Also of concern is whether language and thought are separable or inseparable (Sierpinska, 1994). Merleau-Ponty (1973) is a proponent of the latter position; he believes that thought would become non-existent without language and communication. The relationship between language and thought is extremely complex and conflicting views exist. However, the general consensus (cognitive science) is to presume that thinking is occurring in some language (Sierpinska, 1994).

Vygotsky (1962)

Vygotsky was one of the earliest theorists to begin researching the area of learning and its association with language. He concluded that language is inextricably linked with thought – '..the concept does not attain to individual and independent life until it had found a distinct linguistic embodiment.' (Vygotsky, 1962, p. 4). Although a

thought comes to life in external speech, in inner speech energy is focused on words to facilitate the generation of a thought. If this is the case, it raises an important question – does the nature of the language used affect the nature of the thought processes themselves?

The transition from thought to language is complex as thought has its own structure. It is not an automatic process and thought only comes into being through meaning and fulfils itself in words (Vygotsky, 1962). Thought is mediated both externally by signs and internally by word meanings (Vygotsky, 1962). Communication is only achieved by the thought first passing through meanings and then through words. This raises two questions in relation to the author's research with Gaeilgeoirí. If Gaeilgeoirí cannot interpret the meaning of a given task (due to lack of proficiency in the English language) it will in turn affect their ability to communicate mathematically. Likewise, if a Gaeilgeoir has interpreted the meaning of the mathematical task but lacks competency in the English language to articulate the thought, he may appear to lack understanding of the mathematical task.

It is evident from Vygotsky's research that he supported bilingualism in that he felt the ability to express the same thought in different languages will enable the child to see his/her language as one particular system among many. The child becomes aware of his or her linguistic operation (Vygotsky, 1962). This would support a notion of developing 'balanced bilingualism' (Cummins, 1976). However, the author feels that a number of significant factors need to be taken into consideration in relation to Vygotsky's (1962) theory such as the level of proficiency in both languages, the level of use of both languages and the context in which the languages are used and developed (e.g. home environment vs. school environment), in order for this "self-awareness" to develop.

Sapir (1949)-Whorf (1956) Hypothesis

The basic premise of this hypothesis is that the vocabulary and phraseology of a particular language influences the thinking and perception of speakers of this language, and that conceptions not encoded in their language will not be available to them. Hence, they are proposing that each language will have a different cognitive system and that this cognitive system will influence the speaker's perception of

concepts (Whorf, 1956). Therefore, following this, a Gaeilgeoir should have a different cognitive system to that of an English speaker and this may influence mathematical understanding. It follows from interpretation of this theory that the language we speak facilitates our thinking and perception. If a language restricts a person's thinking then they may fail to develop explanations to problems due to a lack of vocabulary to express the solutions.

Less severe forms of this hypothesis have been developed. One such theory is that language may not entirely shape and determine our thinking but that it may influence it to a certain degree (Sternberg, 2003). Clearly language and thinking interact in a multitude of ways, while also influencing perception and memory. Nature has restricted our ability to use non-linguistic illustrations and thus language is necessary to facilitate mental representation and manipulation (Sierpinska, 1994). This modified version of the Sapir (1949) – Whorf (1956) Hypothesis is one of the key theoretical lenses employed by the author in the analysis of her data.

Bruner (1975)

For Bruner (1975), language, its nature and function, should form part of any theory of cognitive development. He developed three types of mental representations that facilitate understanding:

- Enactive Representations: those that could be mediated through actions.
- Iconic Representations: those that could be mediated through pictures.
- Symbolic Representations: those that could be mediated through symbols or language.

Command of all three representations is necessary for cognitive development and they are developed in the order outlined above. Bruner (1975) emphasises that it is the use of language as an instrument of thinking that is of importance, as well as its affect on cognitive processing (regardless of what type of language).

Language possesses a

"power not only for communication but for encoding 'reality', for representing matters remote as well as immediate, and for doing all these things according to rules that permit us both to represent 'reality' and to transform it by conventional yet appropriate rules."

(Bruner, 1975, p.25/26).

For Bruner (1975), it is clear that language is an essential instrument of thought and is necessary for understanding and combining experiences, and is required for organising concepts.

2.4.2 Language and Understanding

Language plays a key role in developing our understanding. "Understanding can be thought of as an actual or a potential mental experience" (Sierpinska, 1994, p.1). Sierpinska (1994) defines these mental experiences as 'acts of understanding' and an act of understanding is distinguished from 'an understanding', which is the potential to experience an act of understanding. These acts of understanding occur at a particular time and are short in duration. In education, understanding is often correlated with cognitive activity, over a longer period of time. In this 'process of understanding', 'acts of understanding' represent the important steps while the attained 'understandings' represent the supports for further development (Sierpinska, 1994).

For many, understanding is often associated with meaning and/or understanding why (e.g. Piaget, 1978). Understanding can be described in relation to meaning, while meaning can be described in terms of understanding, thus heightening the confusion surrounding the topic. In order to be consistent in explaining the association between meaning and understanding, "the object of understanding is the same as the object of meaning: it is the sign broadly understood." (Sierpinska, 1994, p.23). Therefore, the concept/thought forms the basis of our understanding, while what we seek at understanding are the signs that embody these concepts/thoughts. Because language and thought are interrelated (Bruner, 1975; Vygotsky, 1962) and thought is engaged in our understanding, then language is involved in developing our understanding. Understanding unveils a meaning: learners move from what the text states to what the text is articulating (Sierpinska, 1994).

Sierpinska (1990) identifies four basic mental operations involved in understanding:

- ***Identification:*** referred to in the sense of discovery or recognition of the object of intended understanding. It involves order or hierarchy by classifying objects with related objects.
- ***Discrimination:*** by discriminating between two objects, one identifies the two objects as being different objects. There are several degrees of discrimination ranging from mere perception, comparison to abstract relation.
- **Generalisation:** this is a cognitive operation in which the object of understanding is thought of as a particular case of another situation. Therefore it is necessary to be able to identify the object intended for understanding and act upon it.
- ***Synthesis:*** this involves seeking a common element or link between several generalisations, and their comprehension overall as a certain system, on this foundation.

These operations are sequential in nature in the sense that identification is necessary before discrimination; one cannot generalise until objects have been discriminated between; and without generalisation synthesis cannot take place.

In mathematical problem solving students rely significantly on identifying between the relevant and irrelevant parts of a problem and use words as cues (Dawe, 1983). Thus if Gaeilgeoirí have not developed their proficiency in the English mathematical register, they may not be able to identify and interpret what the problem is asking. This will have repercussions on the remaining cognitive operations of discrimination, generalisation and synthesis and may affect their overall mathematical understanding. The operations outlined above are the basic processes involved not only in understanding and concept construction, but also in thinking in general (Sierpinska, 1994).

2.5 Types of Mathematical Understanding

Several types of mathematical understanding have been identified. In his 1978 article, Skemp discusses the "alternative meanings" associated with mathematical understanding (p.9). The first meaning he refers to is 'relational understanding' which he defines as '..knowing both what to do and why.' (Skemp, 1978, p.9). The alternative to this is 'instrumental understanding' which is described as "..rules without reasons." (Skemp, 1978, p.9). Further elaboration of these concepts followed in his article, in particular their application to teaching and pupil learning. A predominant emphasis on the use and application of mathematical rules and formulas is associated with instrumental teaching and learning. Skemp (1978) empathises with teachers who encourage this type of understanding as it allows for easier and immediate comprehension of material. Also, learners can calculate answers rapidly and the probability of mathematical errors is reduced due to less knowledge being required, thus facilitating immediate reward for correct answers (Davidenko, 2000). In contrast, relational understanding not only requires student comprehension of mathematical rules but also of the relationships between concepts and procedures (Skemp, 1978). This type of knowledge is seen as deeper and more meaningful, yet easier to retrieve when required (Davidenko, 2000). Relational understanding allows students to adapt and apply their knowledge to mathematical tasks that they may not have seen before. More over, this type of understanding is long-term as opposed to immediate and context specific.

Skemp's (1978) concept of instrumental and relational understanding was the catalyst for further investigation into mathematical understanding (Herscovics, 1996). Byers and Herscovics (1977) developed a model of mathematical understanding which "complemented the relational and instrumental modes with intuitive understanding (as evidenced by the solution of a problem without prior analysis of the problem) and formal understanding (as evidenced by the ability to connect mathematical symbolism and notations with relevant mathematical ideas and the ability to combine these ideas into chains of logical reasoning)" (Herscovics, 1996, p.356).

A framework for learning mathematics with understanding was proposed by Hiebert and Carpenter (1992). It is based on a constructivist perspective of learning mathematics in which they perceive learning as "the process of constructing internal representations of information and, in turn, connecting the representations to form organized networks" (p.81). This perspective on learning views understanding as the process of linking previous knowledge with new information, through mental connections. This process permits reflection on, modification and/or the creation of new representations of mathematical concepts (Davidenko, 2000). In their framework, Hiebert and Carpenter (1992) refer to *conceptual* and *procedural* knowledge from this perspective of connections and representations. "Conceptual knowledge is equated with connected networks (….) rich in relationships" (p.78). This type of knowledge is required for understanding the mathematics involved in all stages of a procedure. This type of understanding allows students to adapt/modify the procedures in order to solve new mathematical problems. Thus, "conceptual knowledge extends the procedure's range of applicability" (p.78). Procedural knowledge is defined "as a sequence of actions. The minimal connections needed to create internal representations of a procedure are connections between succeeding actions in the procedure" (p.78). Memorisation of mathematical procedures is commonplace in mathematics education and it can take place without conceptual knowledge. By relying solely on procedural knowledge, students fail to apply and adapt the mathematical procedures to new situations. This is similar to Skemp's (1978) concept of instrumental understanding (Davidenko, 2000).

2.6 Cultural Issues

Language, thinking, learning and understanding cannot be discussed solely from a cognitive perspective. Cultural and pedagogical influences also need to be explored given the central role of language in both of these concepts.

2.6.1 Mathematics as a Cultural Phenomenon

Many researchers/teachers conceive mathematics as being culture free. Bishop (1988) however, viewed mathematics as a cultural phenomenon as mathematics is "conceived as a cultural *product,* which has developed as a result of various activities" (p.179). Mathematics can be perceived as a developing culture within an

overall culture. Given that many different cultures exist, we will have different types of mathematical cultures developing. In different cultures, different mathematical features are focused on (Sierpinska, 1994). For example, some cultures value numbers and counting, hence a large sequence of numbers has been developed and children are encouraged to master this sequence. Whereas other cultures have not developed numerals above a particular number and do not consider numbers as entities in themselves (Sierpinska, 1994). Thus, it may be necessary to take into consideration the cultural background of Gaeilgeoirí and the impact of this upon their transition to English-medium education and the new culture (school, social, mathematical) that they are immersed in.

2.6.2 Pedagogical Concerns

Teaching and learning are central to a culture's functioning. Education is often regarded as a 'transmission of culture' and a method for its 'reproduction' (Bernstein, 1971) and thus influences mathematical teaching and learning, i.e. the culture of the mathematics classroom.

Nickson (1992) identified several factors influencing the culture of mathematics classrooms:

- Perceptions of mathematics as a subject.
- Perceptions of roles within the classroom (teachers' and students').
- The context of the mathematics classroom.

A formalist perception of mathematics views it as consisting of "immutable truths and unquestionable certainty", whereas a cultural perspective views mathematics as "objective knowledge, where knowledge is seen as resulting from competing theories that are proposed, made public, and tested against other theories and held to be true until 'falsified' by a better theory" (Nickson, 1992, p.103). A teacher's perception of mathematics influences their teaching strategies. Those that view mathematics from a formalist perception teach by a means where students are expected to listen and accept that the teacher's knowledge is correct. However, those that view mathematics as being related to human culture and real life experiences employ a constructivist approach to teaching (Davidenko, 2000). This method of teaching positions students

as playing an active role in the construction of their own knowledge, through the means of reflection and communication of mathematical concepts (Noddings, 1990).

The culture of the classroom is influenced by students' perceptions of mathematics and their role in the classroom environment. Most mathematical classrooms involve didactical teaching where the students listen and accept the teacher's mathematical knowledge. Nickson (1992) refers to this as an 'accepting attitude' and is reflective of students' beliefs of mathematics as consisting of immutable truths and the teacher as an authority figure and the source of knowledge. Whereas students who view mathematics from a cultural perspective and have a more positive perspective of their role in the classroom will be more actively involved in their mathematical learning through questioning and discussion (Davidenko, 2000).

2.6.3 Understanding and Culture

"Understanding is both developmentally and culturally bound. What a person understands and how he or she understands is not independent from his or her developmental stage, from the language in which he or she communicates, from the culture into which he or she has been socialized."

(Sierpinska, 1994, p.138)

Education, knowledge and logic are cultural ideas – they are derived from a particular culture while also generating a culture themselves. Culture influences understanding between the mathematics register and the English/Gaeilge register, between teachers and learners, and also between logical thinking and everyday thinking.

Sierpinska (1994) illustrates a mathematical culture as based on Hall's Cultural Triad (1981). In this triad three levels are distinguished between – the 'formal', the 'informal', and the 'technical'- and each has a distinct influence on mathematical understanding. At a 'technical' level the mathematical culture consists of theories and of knowledge that is widely accepted by the community of mathematicians. It consists of logically justified, unequivocal knowledge (Sierpinska, 1994). At the

'formal' level, mathematical understanding is established from a culture's beliefs and values. This includes attitudes towards the subject, as well as perceptions of the subject and its relationship with reality. These can be positive or negative (Sierpinska, 1994). At the 'informal' level, mathematical understanding is established through systems of action and thought. It is the inferred knowledge of methods of approaching and solving mathematical problems. It involves levels of rigour and implicit conventions (Sierpinska, 1994).

Through the 'formal' and 'technical' we obtain certain knowledge about mathematics e.g. algorithms, proofs, problem solving and mathematical theories. This understanding can be used passively (Sierpinska, 1994). It is only through the 'informal' level that we become active learners of mathematics by asking questions, proposing hypotheses and generalisations, synthesising concepts and justifying mathematical understandings (Sierpinska, 1994). These three levels are constantly interacting and it is this characteristic of the mathematical culture that makes change, and thus understanding, possible (Sierpinska, 1994). This triad demonstrates how mathematics understanding is culturally influenced.

2.7 Learning Mathematics in a Second Language

There is growing recognition that language (and bilingualism/multilingualism) plays a key role in mathematics teaching and learning (Barwell, 2009). Given the increase in international migration, the changing status of minority/indigenous groups and the dominance of English as a language for learning and teaching mathematics, many students face a transition to learning mathematics through the medium of English (Barwell, Barton & Setati, 2007). Much diverse research has been undertaken on the effect of second language teaching in mathematics education (Adler, 1998; Barwell, 2009) but this research monograph is specifically concerned with addressing the influence of bilingualism on mathematics education and investigating the difficulties encountered with the English mathematics register when English is the students' second language of learning.

2.7.1 Mathematical English

Bohlmann (2001) highlights the importance of language for mathematical learning given that 'It is the medium by which teachers introduce and convey concepts and

procedures, through which texts are read and problems are solved' (p.6). For an English as an Additional Language (EAL) student, the challenge they face is twofold in that they have to acquire the new language of learning, as well as learning mathematics through the medium of a new language (Bohlmann, 2001). Being proficient in conversational English does not guarantee successful learning in mathematics. As Barton and Neville-Barton (2003) emphasise, proficiency in 'mathematical English' is an important factor in learning mathematics. Naudé's (2004) work supports this view. Her comparative study between Afrikaans students that attended English lectures and Afrikaans students that attended Afrikaans lectures, found that there was no significant difference in performance between the two groups, even though the Afrikaans students attending the English lectures were academically stronger. It suggests that they are experiencing a disadvantage due to not being proficient in mathematical English. Similarly, when examining the influence of language on the mathematical performance of children, Dawe and Mulligan (1997) concluded that teachers need to encourage students to recognise and distinguish between 'mathematical' English and 'natural' English as these are sources of confusion and lead to errors in performance.

In her review of the relationship between mathematics education and language Setati (2002) highlights two important studies carried out in South Africa. At primary school level Rakgokong (1994) found that the use of English as the medium of instruction, where English is not the mother-tongue of the students significantly reduced the students' ability in meaning making and problem solving. When English was the sole language used in the learning environment, students were restricted in engaging in the discourse of the classroom, thus affecting the development of their mathematical knowledge and understanding (Setati, 2002). At third level Varughese and Glencross (1996) found that students encountered difficulties with the English mathematics register, in particular with understanding terms such as integer, perimeter and multiple. These were first year students for whom English was not their first language (Setati, 2002). Thus these studies support the contention that language influences mathematics learning and understanding.

Three important studies have taken place at second and third level education in New Zealand with the primary focus on students learning mathematics for whom English

is a second language (Barton, Chan, King, Neville-Barton & Sneddon, 2005). These studies were stimulated by an interest in investigating the relationship between mathematics and language, and the need to support students in the transition to English-medium education. Findings from the first study indicate that students learning through the medium of a second language (English) have greater than anticipated difficulties with text, and that they wrongly rely more on symbolic modes of working (Barton & Neville-Barton, 2003). The second study had important findings demonstrating that second language mathematics learners were unaware of their disadvantage (Barton, Chan, King & Neville-Barton, 2004). The above studies culminated in the design and implementation of the third study (Neville-Barton & Barton, 2005). The research questions were concerned with the relationship between English language proficiency and mathematics achievement, and the particular features of the mathematics language that cause difficulties for students for whom English is a second language. The findings concluded that students experience a disadvantage, estimated to be between 10 and15 percent, in mathematics as a result of language difficulties. The study confirmed that students are not aware of the difficulties they are experiencing. Diverse language features were identified as sources of difficulties with word order and prepositions the most significant causations, in addition to logical structures (e.g. implication, conditionals and negation). As anticipated, mathematical questions expressed in everyday contexts contributed to the difficulties experienced by the students.

Similarly at third level education Yushau and Bokhari (2005) undertook an exploratory intervention program at King Fahd University of Petroleum & Minerals, with preparatory year mathematics students for whom English was a new language of instruction (subtractive environment). Prior to entry into university the students would have learnt entirely through the medium of Arabic - the first language of the majority of the students. General consensus at the university is that these students perform below expectation, contrary to the level of mathematics ability on selection for entry into the university, and this low performance is attributed to lack of English proficiency (Yushau & Bokhari, 2005). The researchers implemented a mediated teaching approach with the primary focus of providing language support in order to improve the students' understanding in mathematics (providing handouts/overhead transparencies with translation of the important terminology). Findings from the

experiment found that the use of translation encouraged students to connect previous learning with new learning, that there was increased participation in class and improved performance in examinations (Yushau & Bokhari, 2005). The authors found that this approach minimised the language barrier evident in the classroom and due to the improved performance in examinations it demonstrates that there exists a connection between language proficiency and mathematics learning.

A major study was carried out by Gorgorió and Planas in 1997 in the Catalonia region of Northern Spain in relation to mathematics teaching and learning in schools with large numbers of immigrant students. Overall the authors found that "the language practices that learners bring to school inevitably affect how and what they learn." (Gorgorió & Planas, 2001, p.10). Lack of continuity in learning for minority language students in Spain can lead to a variety of problems. These include "disruptive behaviours, 'silent autism' behaviours, absenteeism or cognitive and emotional blockages" (Gorgorió & Planas, 2001, p.13). Specific findings in relation to mathematics include difficulties with understanding everyday words within a mathematical context, false sense of understanding and communication (or lack of it) with the teacher. From a teaching perspective, the authors observed that the linguistic barrier is extended beyond everyday communication and teachers feel that "absence of a common language amongst students is a barrier for their mathematical learning." (2001, p.28). They concluded with the recommendation that more research is necessary with a focus on how mathematical language can be taught and on the relationship between "the 'language of the mathematics class', mathematical language and the process of constructing mathematical knowledge." (2001, p.30).

2.7.2 Comparison of Bilingual and Monolingual Students

A number of studies have compared bilingual students' performance on word problems, by comparing them with monolingual students (e.g. Clarkson, 1991, 1992; Clarkson & Galbraith, 1992; Secada, 1991) or when using different languages (e.g. Adetula, 1989, 1990). These studies are difficult to undertake and Mestre's (1986) study highlights this. He compared groups of bilingual Hispanic students and monolingual students, all undertaking a degree in engineering at university. All students completed a reading test in English, an algebra test and a mathematics word

problem test. No significant difference was found between the monolingual and bilingual students' performance on the algebra test, but on the word problem test the bilingual students were slower and less accurate than the monolingual students. Given the results of these two tests, it implies that the difference may not be attributed to the bilingual students having a lesser ability in mathematics. Also, the vocabulary employed in the word problems was suitable for the bilingual students' level of English language proficiency. This implies that the reading and interpretative demands of the mathematics word problem test was a source of difficulty – 'knowing the vocabulary in a word problem is no guarantee that the mathematical relationships…will be appropriately interpreted [by non-native speakers]' (Mestre, 1986, p.399). This evokes the need to investigate further elements such the syntax and semantics of written mathematical text that may pose problems for EAL students. The purpose of a study carried out by Adetula (1990) was to ascertain the extent to which the language of instruction contributes to performance in mathematics. The findings revealed that children performed better when the mathematical problems were presented in their native language than when presented in English (subtractive bilingualism). Adetula concluded that the teaching-learning process is hindered when students are forced to learn mathematics in a 'foreign' language. This is due to the fact that they have to learn new vocabulary as well as being able to express themselves mathematically in the new language.

2.7.3 Language Switching

Clarkson's (1991) work with Papua New Guinea (PNG) bilingual students confirmed that comprehension errors constitute a large number of the errors made by PNG students (grade 6) when solving mathematical word problems. He argues that competence in the mother tongue, as well as in English plays an important role in the comprehension of mathematical text. Latu (2005) found that Pasifika students' learning of mathematics through the medium of English was hindered by an under developed mathematical discourse in both Tongan and Samoan languages and accordingly their ability to deal with complex mathematical sentences, phrases and mathematical terms. This demonstrates the importance of a student's first language of learning for the transition to English-medium mathematics education. Clarkson's (2007) more recent research concentrated on high ability Australian-Vietnamese bilinguals and their use of language(s) when involved in mathematical problem

solving. He found that the students rely on language switching and thus their competencies in both languages are of importance to how they perform on mathematics problems. When language switching (English to Vietnamese) did occur, it was mainly translating entire problems (as opposed to individual words). This may be as a result of all students having "a well-developed mathematical register in Vietnamese" (Clarkson, 2007, p.209) and suggests that it is more than just vocabulary that plays a significant role in the transition to learning mathematics through the medium of English. This process of language switching appears to be an "unconscious and unplanned" action but that there was a move towards using the primary language of instruction in the classroom (Clarkson, 2007, p.212). However, Clarkson (2007) places an emphasis on the role that the mathematics teacher can play given their knowledge of language switching use by bilingual students in order to enhance their students' mathematical ability.

2.7.4 Culture and Socio-Political Issues

English is the dominant language of education and is perceived as the language of power in Africa, even though it is the language of the minority. Setati and Adler (2000) undertook an extensive review of research projects that had taken place in South Africa. Their focus was on mathematics learning and bilingual education. They found a significant relationship between language proficiency and mathematical achievement. In particular they noted "oral proficiency in English in the absence of mother-tongue instruction was negatively related to achievement in mathematics." (2000, p.245). However, an important aspect the authors highlighted is that the findings cannot just be attributed to the learner's language ability alone. Factors including social, cultural and political issues need to be considered also as they have a significant effect on schooling.

2.7.5 Bilingualism and Mathematics Learning

A study, which was primarily concerned with the effect of instruction in two languages on mathematics learning and achievement of Hispanic college students, was undertaken by Cuervo (1991). The experimental group consisted of 32 Hispanic bilingual students who received bilingual instruction (English/Spanish) in mathematics. 118 students (62 of whom were Hispanic) received instructions only in English and these formed the control group. Both groups were uniformly taught the same mathematical topics/concepts from the same book and sat the examinations on

the same dates. Thus the difference between the groups was the language of instruction and this transferred to the administration of the initial tests (4) in which the bilingual group received both English and Spanish versions. However, both groups received the final examination in English only. The study found that the Hispanic students who participated in bilingual instruction achieved greater academic success than those that received English instruction only. In particular they performed better in the topics of logic, probability/statistics and geometry, but not in algebra. Cuervo (1991) concluded that bilingual instruction (Spanish/English) enhanced Hispanic college students' achievement in mathematics in comparison to instruction in English only.

A study carried out by Jones (1993) draws attention to the advantages associated with being bilingual and mathematics achievement. In particular he highlights the benefit of the development of the mathematics register in a minority language as they tend to be developed relatively recently in comparison to a majority language, and thus the terminology employed tends to avoid linguistic complexity and employs a more self-explanatory mode. This is true for the case of Welsh. Furthermore it has been demonstrated that the way in which numbers and arithmetical relationships are expressed in a language could influence children's understanding (Dowker, 2005). Dowker (2005) is concerned that English speakers may experience a disadvantage in comparison to students learning through another language due to the irregular counting system employed in the English language. In the case of Welsh medium education, the counting system used in schools is completely regular and thus students studying mathematics through Welsh may experience an advantage over students studying mathematics through the medium of English in Wales (Dowker, 2005). This theory is supported by findings that students in Welsh medium education perform better in mathematics state examinations than those in English-medium schools (Reynolds, Farrell & Bellin, 2002). Findings from Dowker's study (2005) include that children studying through the medium of Welsh have better number skills and this continues well into primary school. Also, the study demonstrated that "the effects of language on mathematics, though they are important, are quite specific." (Dowker, 2005, p.29).

French immersion (additive bilingualism) programmes have proven very popular in Canada and participation has increased significantly over the years (Bournot-Trites & Tellowitz, 2002). This is perhaps due to the positive findings of studies carried out in Canada that support second language instruction and learning (Cummins & Swain, 1986). Bournot-Trites and Reeder (2001) found that the group with high intensity French instruction performed better than the comparison monolingual groups using standardised tests in mathematics and science. A similar study was carried out by Swain (1996) and total immersion students performed as well as their monolingual peers, whereas partial immersion (some of the subjects taken through the medium of French, the remainder through English) students often did not. This was attributed to a lack of proficiency in the second language (French). As part of a large-scale report to the Ontario Education Quality and Accountability Office (EQAO), Turnbull, Hart and Lapkin (2000) assessed the English and mathematics performance of French immersion students. They concluded that immersion students performed at a comparable level with English programme students by Grade 3. By Grade 6 they outperformed their monolingual counterparts in all skill areas. Therefore, overall Canadian students have experienced positive benefits from participating in French immersion programmes.

2.7.6 The Influence of Mother-tongue

When examining the Chinese language for learning early number concepts, it was found that Asian language structures are very different from English language structures, thus causing problems for Asians studying mathematics through English in America (Geary, Bow-Thomas, Liu, & Stigler, 1996). However, Geary et al. (1996) did note that the language structure of Asian number names assisted Chinese children in developing early number concepts in comparison to English language structures. This supports his viewpoint that it is beneficial for the learner to maintain their mother-tongue language throughout the student's mathematical learning experience (Geary et al., 1996). Han and Ginsburg (2001) undertook a study of Chinese, English and "Chinglish" (Chinese words translated into English) mathematical words. More Chinese mathematical words are rated clearer than are English mathematical words by qualified judges. Chinglish mathematical words tend to be rated clearer (more comprehensible) than English words also. Overall they found that the inherent compound word structure of the Chinese language seems well

suited to convey mathematical concepts. In the second part of the study, the authors investigated the relationship between clarity of words, students' Chinese reading ability and achievement in mathematics. A strong relationship was found between reading ability and attainment in the mathematics tasks containing words rated clear by the judges. They contribute these findings as factors in the superior performance of students studying mathematics through the medium of Chinese as opposed to English. Thus it is more beneficial for Chinese students to study mathematics through the medium of their mother-tongue.

Structural differences between the English and Mandarin languages are factors responsible for differences found in the performance of third level Australian business studies students (Galligan, 1995). Three tests were carried out, each in the students' first language and they involved pure calculations, context free word problems and word problems in context. Students achieved similar results on the mechanical problems, but the word problems caused difficulties for non-English background students, thus highlighting their language difficulties. In Galligan's (2001) review of differences between the English and Chinese language, she found that large differences in the syntax, semantics, orthography and phonetics exist between the two languages. For example, Chinese noun phrases tend to be left-embedded and this may affect cognitive processing. But with respect to English, the cognitive processing load is greater as the reader must remember the descriptive clause before dealing with the sentence. Accordingly, the easier syntax and better structured word order may facilitate access to the target question (Galligan, 2001). There is little use of the passive voice in Chinese mathematical texts, whereas in English mathematical text passives are common. It is expected that passive voice requires more processing than active voice (Galligan, 2001). The use of compound words in Chinese mathematical texts helps describe the concept, as opposed to labelling in English e.g. the word for 'diameter' in Chinese when translated to English means straight line. Naturally this lends itself to a better understanding of mathematical concepts (Galligan, 2001). Subsequently some of the differences outlined in her review may have consequences for the processing of mathematical text.

2.8 Conclusion

Studies such as Dowker (2005) and Neville-Barton & Barton (2005) demonstrate that the mathematics registers in Gaeilge and in English will play a significant role in the transition from Gaeilge-medium to English-medium education. Thus the author needs to look at the particular aspects of the English mathematics register that may be sources of difficulty for Gaeilgeoirí in the transition to English-medium mathematics education at second and third level education in Ireland.

By delving into the area of psycholinguistics the complex nature of language and its relationship with thinking and understanding is revealed. Although conflicting views exist, the general consensus (cognitive science) is to presume that thinking is occurring in some language (Sierpinska, 1994). Language is necessary for comprehending and combining experiences, and is required for organising concepts. Concepts and thoughts are foundations of our understanding. At the same time what we are trying to understand are the signs that represent these concepts and thoughts. Language and thought are interrelated (Bruner, 1975; Vygotsky, 1962) and thought is necessary for our understanding, then language is involved in developing our understanding. The Sapir (1949)-Whorf (1956) Hypothesis is also employed when looking at the data. The basic premise of this hypothesis is that the vocabulary and phraseology of a particular language influences the thinking and perception of speakers of this language, and that conceptions not encoded in their language will not be available to them. A less severe form of this hypothesis has been employed by the author in that she supports the premise that language may not shape and determine our entire mathematical thinking, but that it may influence it to a certain degree (Sternberg, 2003).

One cannot overlook the importance of culture and its influence on mathematical teaching, learning and understanding. This is heightened for this project in the sense that it is concerned with mathematical learners moving between cultural environments. These include from the culture of primary to secondary education, and secondary to third level education; from a culture of Gaeilge-medium learning to a culture of English-medium learning; pedagogical influences on the mathematical culture and the development of mathematical understanding.

Clearly there are a number of significant theories unfolding throughout this Chapter and these informed the design of the research project undertaken and the interpretation of the key findings emerging (Chapters 4 and 5).

3.

Bilingual Education and Bilingualism

3.1 Introduction

This study is concerned with bilingualism and its influence on mathematics education in an Irish context. This Chapter sets out to examine definitions of bilingualism, cognitive theories of bilingualism, and the various forms of bilingual education that exist. The concepts and theories presented here are utilised for examining the issue of bilingualism and mathematics education in Ireland. Given that no research into bilingualism and mathematics education has been undertaken in Ireland it is necessary to draw on international literature in order to inform the study and methodological practices employed in this investigation. An important outcome of this review is the author's ability to classify Gaeilge-medium schools (Gaeltacht schools or Gaelscoileanna/Gaelcholáistí) as promoting Additive or Subtractive bilingualism.

3.2 Bilingualism

Defining bilingualism is difficult, in particular defining whether a person is bilingual or not (Baker & Prys Jones, 1998). The majority of the definitions include reference to an ability or proficiency to speak two languages. This is where variation is encountered, and a spectrum of definitions is created (Romaine, 1989). At one end of the spectrum is MacNamara's (1966) definition. He requires only minimal ability in one or more of the language skills (reading, writing, speaking and listening) in the person's second language. At the other end of the spectrum is the approach by Bloomfield (1933, p.56) who specifies "native-like control of the two languages" as the measure of bilingualism. Grosjean and Moser-Mercer (1997, p.165) developed the notion of a "complementarity principle" in which they emphasise that bilinguals use their languages for different purposes and in different domains of life. Dominance in

one language over the other is common among bilinguals depending on the use and function of each language, but little progress has been made in measuring the exact degrees of bilingualism evident. Similarly, studies involving bilinguals tend to focus on only one language, but due to the complex nature of the issue of bilingualism, aspects of both languages need to be taken into account. Grosjean (1998) highlights the vital importance for researchers in the field to specify what knowledge bilingual subjects have of their respective languages, because contradictory findings can be attributed to the use of different interpretations of 'bilingual'. Terms such as "balanced bilingual" and "semi-lingual" have been coined but these are vague and lack precision, primarily because of the difficulty and lack of valid instruments available to measure bilingual proficiency. Thus there is a need to develop precise definitions and measurements of bilingual proficiency. However, bilingualism is more than just possessing an ability to use two languages. Bilingualism is a consequence of educational, social, economic, cultural and political struggles (Baker, 2000). For example, a Government plays a major role in deciding an official language(s) of a country and what language(s) is to be used as a medium of learning within the education system. Similarly, the economic and cultural status of a language will determine its perceived value within an education system.

3.3 Second Language Acquisition

Understanding second language acquisition as a process highlights the complexity of bilingualism, yet it is necessary in order to determine how students become bilingual. The central issue is "Who learns how much of what language under what conditions?" (Spolsky, 1989, p.3). The **'who learns'** is concerned with individual differences and is constantly changing. The **'how much of what language'** part is concerned with the specific language skills that are being developed, the measures used to assess this skill development, as well as cultural influences. The **'under what conditions'** element focuses on situation and context i.e. the learning environment and the strategies influencing language learning. Given the huge diversity that exists between learners and in learning a language, the following frameworks and theories have been proposed by researchers in the field as guidelines for those working with students learning through the medium of a second language, and to draw attention to prevalent issues influencing their learning.

Some prominent theories of second language acquisition exist such as Ellis' Framework (1985), Lambert's Model (1974), and Gardner's Socio-Educational Model (1985). Although variations exist between the different models, what is clear is that consideration needs to be given to individual characteristics (e.g. ability, motivation), the type of language skills being learnt (e.g. grammatical, conversational), context and situation variables (e.g. school type attended), and cultural variables when analysing second language acquisition. However, this research project is concerned specifically with bilingualism and mathematics learning/understanding. Lambert (1974) distinguishes between two second language acquisition outcomes – Additive and Subtractive. Additive bilingualism occurs when a second language and culture have been acquired without loss or displacement of an individual's first language and culture, and a positive self-concept is correlated with this form of bilingualism (Baker, 1996). Subtractive bilingualism occurs when an individual's first language and culture are replaced by the new language and culture, usually occurring in a pressurised context. As a consequence a negative self-concept may develop due to loss of culture and identity (Baker, 1996). These Additive and Subtractive outcomes have been affiliated to educational contexts and the author believes that the school type attended by Gaeilgeoirí (i.e. a Gaeltacht school or Immersion Education) is significant and affects the findings emerging from the data collected. Naturally, the concern is with how these types of bilingualism may affect the mathematical learning and understanding of Gaeilgeoirí participating in the study.

3.4 Types of Bilingual Education

The term "bilingual education" is deceiving on first glance. One would assume that it simply implies teaching and learning through the medium of two languages. Baker and Prys Jones (1998) reiterate how complex the expression actually is and that a set of questions needs to be addressed in order to assess the learning context.
These questions include:

- Are both languages used in the classroom?
- For how long are the languages being used in school?
- Are two languages used by all or some students?

- Are two languages used by the teachers or just by the students?
- Is the aim to teach a second language or to teach through a second language?
- Is the aim to support the home language or to move to an alternative majority language?

(1998, p.464).

There are several different types of bilingual education programmes and within these types further subdivisions arise. These programmes have developed over the past four decades, mainly in the United States where a variety of second language contexts exist. These programmes can be classified under the broad terms of weak or strong bilingual education (Baker, 2000). Weak forms tend to include schools/institutions that contain bilingual individuals as opposed to encouraging bilingualism. These schools/institutions usually enrol language minority students with the aim of developing learning through the majority language (Baker & Prys Jones, 1998). Development of the home culture and language is not fostered. On the other hand, strong forms of bilingual education are developed when the primary aim is to develop complete bilingualism in both languages, and both cultures are supported (Baker & Prys Jones, 1998). Naturally, the type of bilingual programme implemented is reflective of cultural beliefs in that society and will impact upon the learning outcomes of the students. A classification system for bilingual education programmes has been developed by Baker and Prys Jones (1998) and is summarised in Table 3.1.

It is important to note that these are general classifications and it is not assumed that all bilingual education contexts can be classified under one of the above. However, from analysis of the Irish education system, three obvious contexts are emerging –

Submersion:	Students from Gaeltacht schools and Gaelscoileanna/Gaelcholáistí transferring to English-medium education.
Immersion:	Students in Gaelscoileanna/cholaistí.

Maintenance Heritage Language: Students from a Gaeltacht area attending a school in that Gaeltacht.

For this research project the author is primarily concerned with the Submersion context – students transferring from learning mathematics through the medium of Gaeilge to learning mathematics through the medium of English (see Fig. 3.1). This will involve students at second level and third level education. Since two types of Gaeilge-medium education exists at primary and secondary level in Ireland (Immersion and Maintenance Heritage Language), it is anticipated that the previous learning environments of the students entering the submersion contexts may be of significance. Thus it will be necessary to investigate both types of Gaeilge-medium education and to establish if a relationship exists between the type of Gaeilge-medium education attended and the students' achievement in mathematics in an English-medium education context.

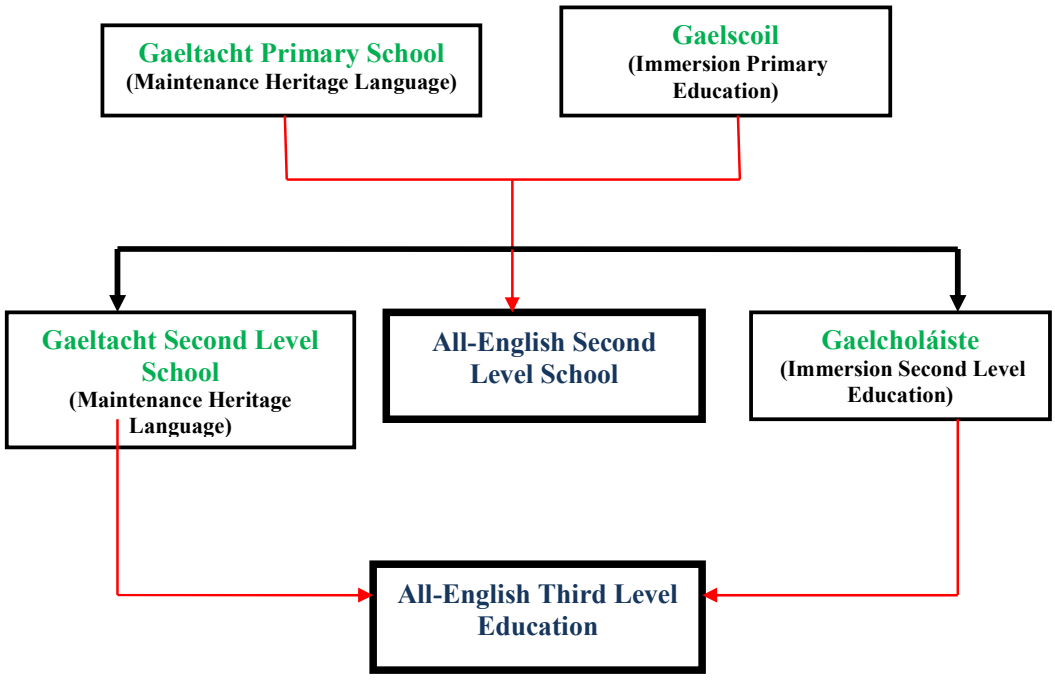


Figure 3.1 Transitions being investigated in the study

3.5 Additive and Subtractive Bilingualism

Furthermore the concept of weak and strong bilingual education programmes is associated with ***subtractive and additive bilingualism***. Subtractive bilingualism usually occurs when a majority language replaces a minority language (Lambert, 1990). Students are forced to adapt quickly into mainstream education where the majority language is used as the medium of instruction. No support is given to the mother-tongue, resulting in it slowly being replaced (Bournot-Trites & Tellowitz, 2002). This is similar to the experiences of Gaeilgeoirí transferring from Gaeltacht schools (Maintenance Heritage Language) to all-English-medium schools. The situation is different in the case of additive bilingualism. In this instance the mother-tongue of the child is the majority language and they opt to study through a second language, which is a minority language (Lambert, 1990). This is similar to Immersion education (Gaelscoileanna/Gaelcholáistí) in operation in Ireland. Both languages are supported throughout the schooling and the intention is not to replace the majority language but to develop the second language (Bournot-Trites & Tellowitz, 2002).

Table 3.1 Weak and strong forms of bilingual education

WEAK FORMS OF EDUCATION FOR BILINGUALISM

Type of Program	Typical Type of Child	Language of the Classroom	Societal and Educational aim	Aim in Language Outcome
1.SUBMERSION (Structured Immersion)	Language Minority	Majority Language	Assimilation	Monolingualism
2.SUBMERSION (Withdrawal classes/ Sheltered English)	Language Minority	Majority Language with 'pull-out' lessons.	Assimilation	Monolingualism
3.SEGREGATIONIST	Language Minority	Minority Language (forced, no choice)	Apartheid	Monolingualism
4.TRANSITIONAL	Language Minority	Moves from minority to majority language	Assimilation	Relative Monolingualism
5.MAINSTREAM (With foreign language teaching)	Language Majority	Majority language with L2/FL lessons	Limited Enrichment	Limited Bilingualism
6.SEPARATIST	Language Minority	Minority language (out of choice)	Detachment/ Autonomy	Limited Bilingualism

STRONG FORMS OF EDUCATION FOR BILINGUALISM

Type of Program	Typical Type of Child	Language of the Classroom	Societal and Educational aim	Aim in Language Outcome
7.IMMERSION	Language Majority	Bilingual with initial emphasis on L2	Pluralism & Enrichment	Bilingualism & Biliteracy
8.MAINTENANCE HERITAGE LANGUAGE	Language Minority	Bilingual with emphasis on L1	Maintenance, Pluralism & Enrichment	Bilingualism & Biliteracy
9.TWO-WAY/ DUAL LANGUAGE	Language Minority	Minority & Majority	Maintenance, Pluralism & Enrichment	Bilingualism & Biliteracy
10.MAINSTREAM BILINGUAL	Language Majority	Two majority language	Maintenance, Pluralism & Enrichment	Bilingualism & Biliteracy

[* L1 – First Language * L2 – Second Language * FL – Foreign Language]

(Baker & Prys Jones, 1998, p.470)

3.6 Cognitive Theories of Bilingualism

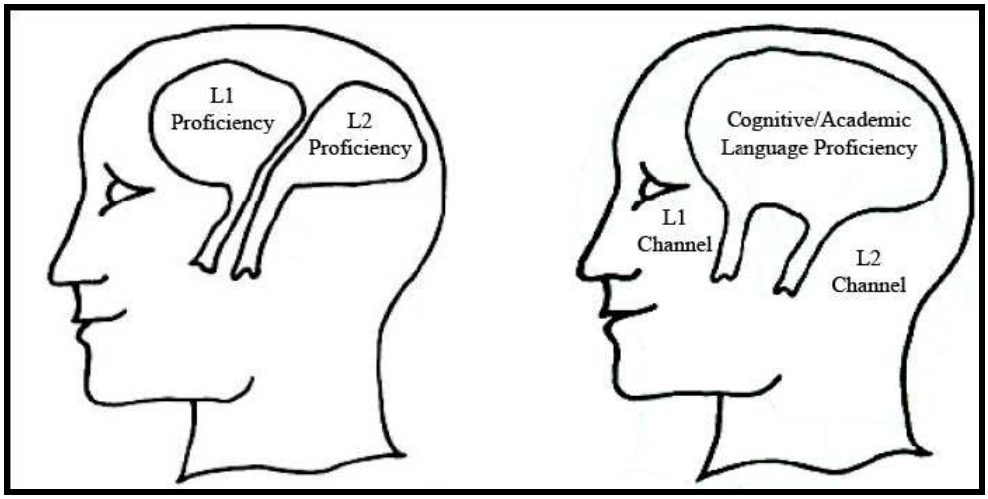
"Some people would argue further that language is somehow related to thinking, learning and cognitive development." (Stubbs, 1976, p.14).

Cognitive theories emerged at the beginning of the twentieth century and have matured and grown into applicable theories. Applications of these theories have focused considerably on language acquisition and learning concerns. Misconceptions about how the brain stores language have led to negative perceptions of bilingualism. The most prominent being that bilingualism may result in "cognitive overload" and thus disadvantage the learner (May, Hill & Tiakiwai, 2004). In the following sections the author discusses a number of theories and theorists who have had a considerable impact on the areas of learning and language.

3.6.1 Separate and Common Underlying Proficiency (SUP/CUP)

The misconception of bilingualism alluded to above is associated with a narrow perception of the mind and its storage of language(s) and is described as the Separate Underlying Proficiency (SUP) model. This misconception views the two languages as being stored separately and independent of one another (Baker, 2001; Baker & Prys Jones, 1998). Analogies include that of the brain having two separate balloons with a restricted amount of space for the balloons (each language) to expand. Or, as a set of scales – an increase in the "weight" of one of the languages will result in an imbalance and loss of a portion of the other language (Baker & Hornberger, 2001).

This model is not an accurate reflection of the working of the mind. A large body of research demonstrates that when abilities in both languages are continued and developed throughout schooling, learners develop a deeper understanding of language and its functions (Cummins, 2002). The assumption that languages are separated in the mind is also false (Baker, 2001, Fig.3.2.).Take for example students who learn mathematical operations (addition, subtraction, multiplication, division) in Gaeilge they will also be able to perform the same operations in English. Therefore interplay exists between both languages.



Separate Underlying Proficiency ***Common Underlying Proficiency***

Figure 3.2 Separate and Common Underlying Proficiencies

(Baker and Hornberger, 2001, p.131-132)

The Common Underlying Proficiency (CUP) is a more apt description of language construction within the mind (Cummins, 1980). The CUP model is depicted in the form of two icebergs, which are separate above the surface. This represents the fact that outwardly both languages are different in conversation. However, underneath the surface, both languages are merged so that they do not function independently of one another (Baker, 2001; May, Hill & Tiakiwai, 2004, Fig. 3.3). The storage of both languages occurs in this area (beneath the surface) and it acts as a central processing unit that both languages contribute to, access and use (Baker, 2001; Baker & Prys Jones, 1998).

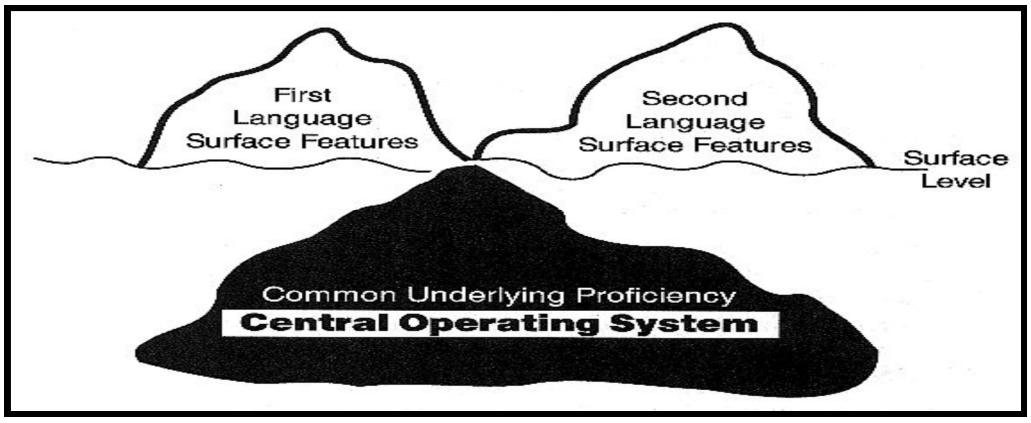


Figure 3.3 Model of Common Underlying Proficiency

(Baker, 2001, p.165)

According to Baker (2001, p.165-166) the CUP model of bilingualism may be summarized in six parts:

1. Irrespective of the language in which a person is operating, there is one integrated source of thought.
2. Bilingualism and multilingualism are possible because people have the capacity to store easily two or more languages. People can also function in two or more languages with relative ease.
3. Information processing skills and educational attainment may be developed through two languages as well as through one language. Both channels feed the same central processor.
4. The language the child is using in the classroom needs to be sufficiently well developed to be able to process the cognitive challenges of the classroom.
5. Speaking, listening, reading or writing in the first or the second language helps the whole cognitive system to develop. However, if children are made to operate in an insufficiently developed second language (L2) in a subtractive bilingual environment (as occurs for many bilingual students in English-language-only classes), the system will not function at its best. If children are made to operate in these classroom contexts, the quality and quantity of what they learn from complex curriculum materials, and produce in oral and written form, may be relatively weak and impoverished.
6. When one or both languages are not functioning fully (e.g., because of an unfavorable attitude to learning through the second language, or pressure to replace the home language with the majority language), cognitive functioning and academic performance may be negatively affected.

Therefore, given that *both* languages are dependent on one another, this fact needs to be taken into account when investigating Irish bilinguals and their learning of and understanding of mathematics. One cannot investigate one language without examining the other language also and it is necessary to utilize a framework for investigation that reflects this view best.

3.6.2 Cummins (1976) – 'Threshold Hypothesis'

This hypothesis states that the level of mother tongue (L1) proficiency already reached by a student determines if he/she will experience cognitive deficits or benefits from learning in a second language (L2) (Cummins, 1976). This implies that there is a certain 'threshold' that one must reach in their first language before the benefits of studying in a second language can develop. For those who begin studying in a second language before achieving this level, there will be serious learning difficulties and repercussions (Cummins, 1976). Cummins further developed his theory and claimed that there is a threshold for the second language also, which must be achieved so as to "allow the potentially beneficial aspects of second language learning to influence a student's cognitive and academic functioning." (1979, p.222).

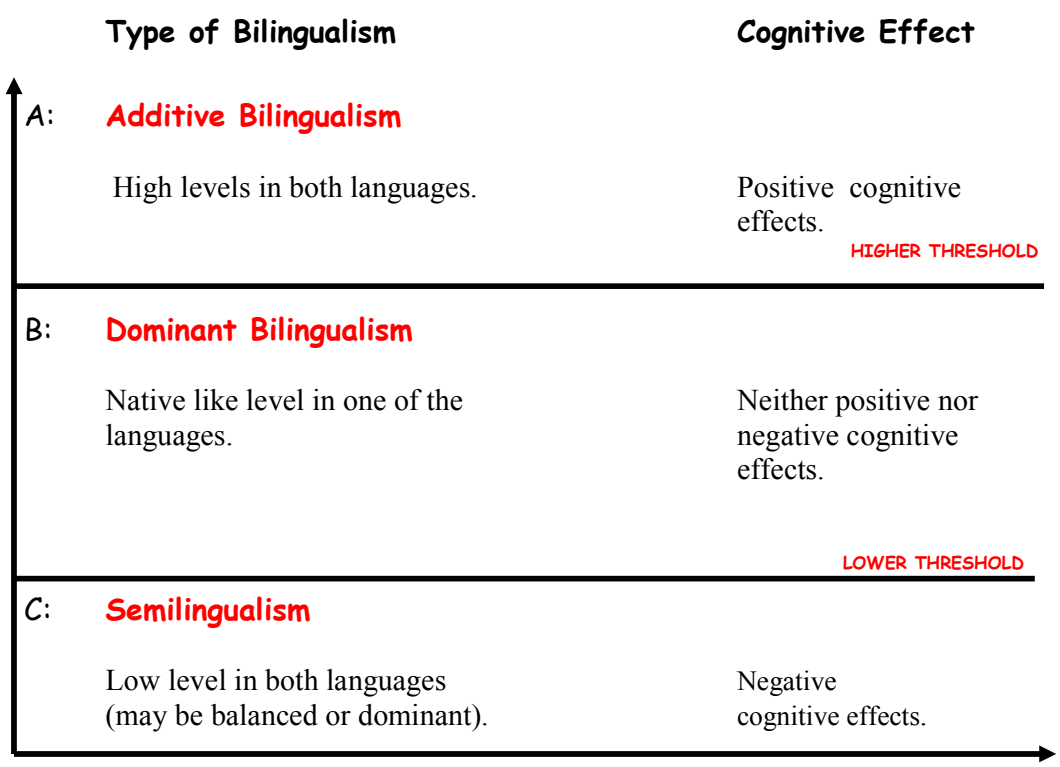


Figure 3.4 Threshold levels and cognitive effects of different types of bilingualism (Dawe, 1983, p.334).

In order to avoid negative consequences of bilingualism it is necessary to reach the first threshold. By reaching the second threshold a bilingual student should experience positive benefits from learning in a second language (Baker, 1996). An

important inference of Cummins' Threshold Hypothesis is that students who are not sufficiently fluent in either of the two languages that they use tend to experience difficulty in mathematics (Minami & Ovando, 2001). At the first level of this Hypothesis the bilingual student has a low level of proficiency in both of the languages and there will be negative cognitive affects for the student's learning (Baker, 2001). At the middle level, the bilingual child will have age-appropriate proficiency in one of their languages (comparable to a monolingual child) but not in both. This dominance in one of the languages is unlikely to influence cognition in any significant positive or negative way (Baker, 2001). The third or top level of this Hypothesis encompasses well-developed bilingual students who have age-appropriate proficiency in both languages and are likely to demonstrate cognitive advantages over monolingual or weaker bilingual students (Baker, 2001).

Therefore, for example, this implies that students, who have learnt through the medium of Gaeilge but have not developed their language to a sufficient level, will experience difficulties when transferring to learning through the medium of English. Gaeilgeoirí who have an appropriate level of proficiency in Gaeilge, but not in English, may not experience any cognitive advantages when learning mathematics through the medium of English. A more positive aspect is that those who have reached the 'threshold' in both Gaeilge and English should experience positive cognitive benefits in their learning. Given that both languages are interdependent and proficiency in both is importanct for cognitive performance, the languages cannot be looked at in isolation as suggested by the SUP model. Clearly the CUP model is consonant with Cummins' Threshold Hypothesis (1976), which reflects the realities of bilingual contexts while being supported by empirical research (e.g. Dawe, 1983; Clarkson, 1992).

The significance of this theory is twofold (May, Hill & Tiakiwai, 2004). Primarily it seeks to establish why minority students under-perform academically when submerged in a school environment where they are learning through the medium of a second language, their weaker language. Secondly it demonstrates how learning through the medium of a minority first language does not appear to hinder the development of the majority language, and it may actually have positive cognitive benefits (Cummins, 2000). Baker, (1996) has developed a three-storied house

analogy in order to demonstrate the bilingual linguistic requirement at each level (Fig. 3.5). Ideally a bilingual student needs to progress beyond the second level in order to attain cognitive benefits from learning in a second language.

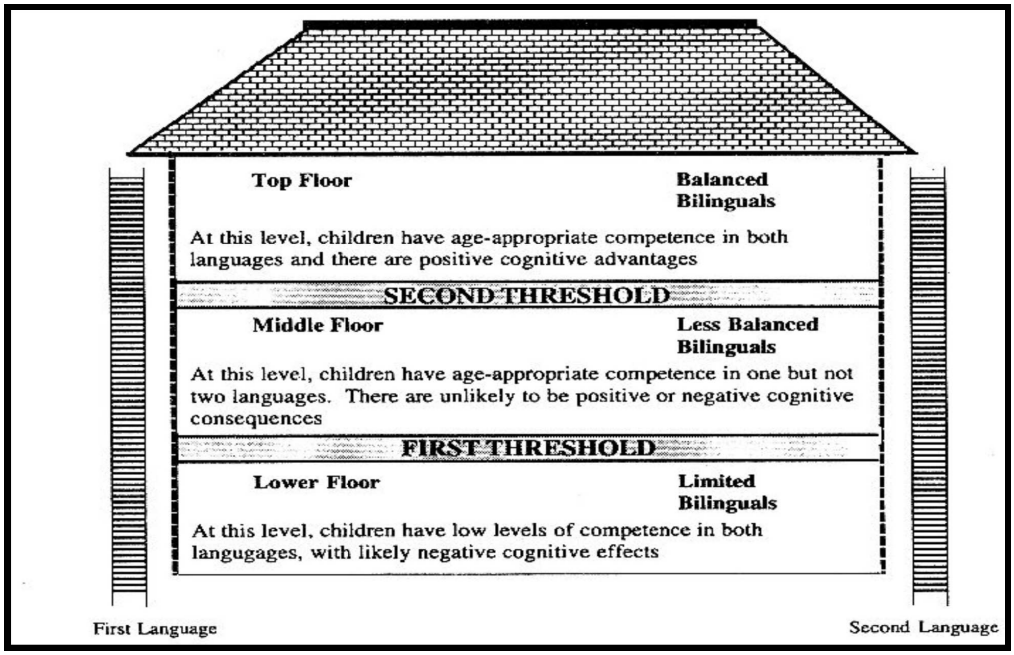


Figure 3.5 Bilingual linguistic requirements (Baker, 1996, p.149)

Although the Threshold Hypothesis appears to vindicate the dissimilar findings in bilingual education, there are a number of weaknesses that need to be addressed. There have been criticisms of Cummins' theory because it cannot be supported experimentally since there is no definition for the "threshold level necessary" (Ahmed, Marriot & Pollitt, 2000, p.21). In this study clear threshold levels were identified for both Gaeilge and English language proficiency at each transition in Irish education, akin to the method employed by Clarkson (2007). A prominent criticism relates to the terms used to describe the various bilingual proficiency levels within the theory which include 'semilingualism', 'dominant' and 'balanced' bilingualism. In particular the term 'semilingualism' has been criticised for the notion of deficit that is implied (MacSwan, 2000) and consequently the term has been disregarded by Cummins (2000). It has also been argued that the use of these terms reflects a narrow view of language competence (Romaine, 1989), and accordingly a stagnant perception of language and of the variation of language use.

However, Cummins (2000) has defended these terms as being reflective of educational contexts e.g. schools that employ two languages of instruction, and that these contexts influence the development of bilingualism. For example, one language may be used more than the other thus resulting in 'dominant' bilingualism in one of the languages. Geva and Ryan (1993) argue that individual difference in intelligence and memory span may also be involved in the transfer of cognitive abilities from one language to the other. Similarly Hoffman (1991) questions how one can measure and define 'educational success', and that reliance on traditionally measured school tests neglects factors such as motivation, attitudes, social issues, schooling, parental support, etc. which are important when determining educational success. Setati (2002) argues that academic achievement is influenced by a number of inter-related factors and that performance cannot be attributed to the degree of language proficiency alone. Consideration needs to be given to "..wider social, cultural, and political factors that infuse schooling." (Setati, 2002, p.7). Clearly, the issues raised are concerned with terminology and lack of detail but significant studies have been undertaken that provide strong support for the Threshold Hypothesis (e.g. Bialystok, 1988, Clarkson, 1992; Dawe, 1983; Lasagabaster, 1998; Mohanty, 1994). These studies provide an explanation of the variation amongst bilingual students and although the theory is controversial in nature, it has influenced educational policies in the USA and in the UK (Yushau & Bokhari, 2005).

3.7 The Developmental Interdependence Hypothesis (1979)

In 1979 Cummins refined his Threshold Hypothesis and this led to the development of his Developmental Interdependence Hypothesis, which had a more in-depth focus on the relationship between a student's two languages. The Interdependence Hypothesis proposed that the level of proficiency already achieved by a student in their first language would have an influence on the development of the student's proficiency in their second language (Baker, 2001). As Cummins (1979, p.233) states:

"...the level of L2 competence which a bilingual child attains is partially a function of the type of competence the child has developed in L1 at the time when intensive exposure to L2 begins:...[an] initially high level of L1 development makes possible the development of similar levels of competence in L2. However, for children whose

L1 skills are less well developed in certain respects, intensive exposure to L2 in the initial grades is likely to impede the continued development of L1."

Therefore, the greater the level of proficiency achieved in the first language will allow for a better transfer of skills to the second language. This hypothesis stemmed from a proposal by Oller (1979) that proficiency in all language skills (listening, speaking, reading and writing) were derived from a single dimension of language proficiency. This was not in line with Cummins' philosophy, which proposed that language proficiency was multi-dimensional.

3.8 Basic Interpersonal Communicative Skills (BICS) vs. Cognitive Academic Language Proficiency (CALP)

These are two individual registers that bilingual students have to develop and accomplish in their first and second languages. Basic Interpersonal Communicative Skills (BICS) relates to communication skills and conversational competence. It relies on phonological, syntactic and lexical skills required to function in everyday contexts – the majority of the time these contexts are cognitively undemanding and contextually supported (May, Hill & Tiakiwai, 2004). Competence in BICS in a second language is achieved within 1-2 years (Cummins, 2000).

On the other hand Cognitive Academic Language Proficiency (CALP) is required for context reduced academic situations. CALP demands manipulation of the surface features of a language in impersonal contexts (May, Hill & Tiakiwai, 2004). The skills required are higher order in nature such as analysis, synthesis and evaluation. Cummins (2000) argues that these skills are a prerequisite for CALP as they provide students with the facility to use language as an instrument of thought in problem solving and this justifies the assertion that it takes 5-7 years for learners to acquire academic language proficiency in a second language. This is further complicated by the fact that Gaeilgeoirí not only have to develop proficiency in the academic register in English but also learn new mathematical content in that language.

Baker (1996, p.151) draws on an appropriate mathematical example to demonstrate the difference between BICS and CALP.

"A child is given a mathematical question such as: 'You have 20 dollars. You have 6 dollars more than me. How many dollars do I have?' At the higher CALP level, the child will conceptualize the problem correctly as 20 minus 6 equals 14. At the BICS level, the word 'more' may be taken to mean 'add-up' with the child getting the wrong answer of 26. The BICS child may think of 'more' as used in basic conversation. However, in the mathematics classroom, this illustration requires 'more' to be understood by the mathematical phrasing of the question."

The Iceberg analogy can be used to demonstrate the underlying concepts of BICS and CALP. Language skills such as comprehension, speaking, pronunciation, vocabulary and grammar lie above the surface and are used in conversations (BICS). Below the surface lie the academic language skills such as analysis, synthesis and semantic meaning (Baker, 2001).

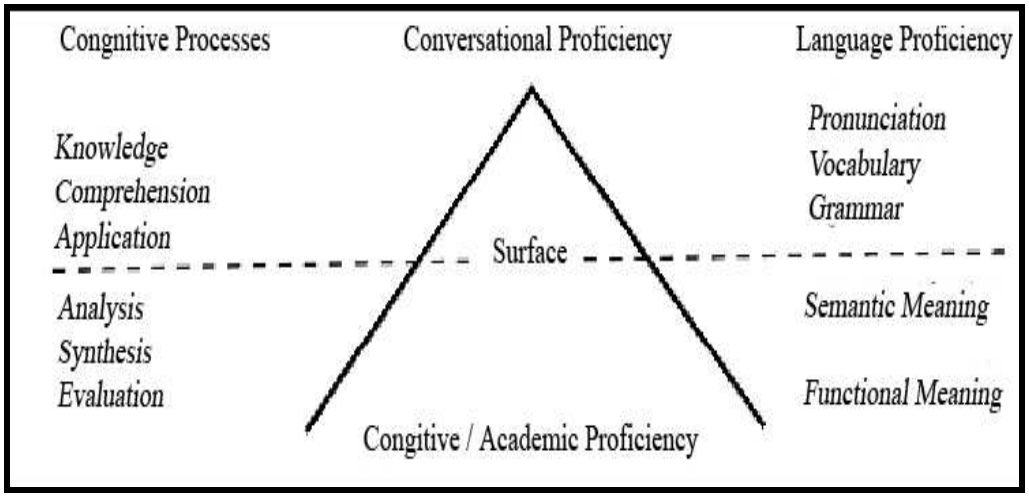


Figure 3.6. Language and cognitive skills required for BICS and CALP

(Baker, 2001, p.170)

According to Cummins (1979), in order for bilingual students to master the academic language proficiency of their second language, their Common Underlying Proficiency must be well developed. This CUP can be developed via the Interdependence Hypothesis, and, can be developed through the first or second

language (additive environment). What is important to note here is that, while second language learners may pick up oral proficiency (BICS) in their new language of learning in as little as two years, it may take up to seven years to acquire the decontextualised language skills (CALP) necessary to function successfully in a second language classroom. Mathematics is located within this CALP and in order for Gaeilgeoirí to attain mathematical academic language proficiency, their CUP must be well developed (Cummins, 1979). This underlying ability in turn can be advanced through the Developmental Interdependence Hypothesis and, depending on the type of schooling, either through a student's first (Gaeltacht schools) or second language (Gaelscoileanna/Gaelcholáistí). Once again there are a number of criticisms of the distinction between language registers, in particular that the differentiation underestimates the demands of conversational proficiency, while overemphasizing the demands of academic proficiency. Also, a potential deficit may be associated with students who do not acquire academic proficiency (Fredrickson & Cline, 1996). However, the author strongly feels that the distinction between language registers facilitates an explanation of bilingual students' relative success/failure when they encounter a new language of instruction in educational contexts.

3.9 Implications for Teaching

Distinguishing between BICS and CALP has implications for teachers of second language/minority language learners. As a consequence, Cummins extended his model so as to facilitate teachers in their design and implementation of programmes to cater for these students. The model is two-dimensional in relation to cognitive demand and contextual embeddedness (Baker, 2001; Cummins, 1981).

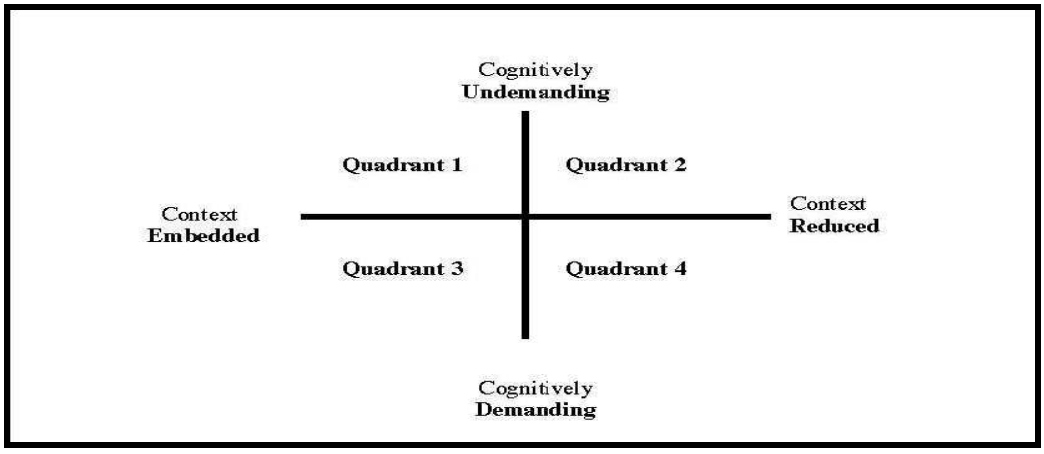


Figure 3.7 BICS and CALP's implications for teaching (Baker, 1996, p.153)

The first continuum (horizontal) is the level of contextual support available to the learner. At one extreme is the context-embedded – informal, face-to-face communication and learners can actively negotiate meaning, and the language is supported by paralinguistic and situational clues (May, Hill & Tiakiwai, 2004). In the context-reduced extreme, learners have to rely mainly on linguistic cues. The latter reflects classroom situations (Cummins, 2000). The second continuum reflects the amount of cognitive involvement necessary for particular situations/activities. Communicative tasks in the upper continuum require minimal cognitive involvement as the linguistic tools required have already been developed and thus can be applied automatically. Tasks in the lower end of the continuum require language skills that have to be developed and therefore are more cognitively demanding. Often, in a learning environment, students are required to organize their language production consciously, while dealing with new and perhaps difficult concepts (May, Hill & Tiakiwai, 2004). Many classroom activities are located in the fourth quadrant and such activities are demanding, in particular for students learning through the medium of a second language.

Therefore, Cummins' theory is suggesting that bilingual students will achieve success only when they have developed an appropriate level of language proficiency in the language of instruction so that they can cope with the context-reduced, cognitively-demanding situations that arise in learning environments (Fourth Quadrant, Fig. 3.7). On the other hand, learners working at a context-embedded level may be hindered in developing their understanding of the content of the lesson, while also failing to develop high order cognitive processes (May, Hill & Tiakiwai, 2004). Teachers of bilingual learners must avoid making the assumption that their learners are proficient in the language of instruction if the learners demonstrate conversational competence. Teachers must acknowledge that bilingual students experience greater difficulties in acquiring the academic language proficiency in the language of instruction and appropriate teaching strategies and interventions need to be implemented in order to facilitate bilingual students' understanding of mathematics.

3.10 Conclusion

The literature proposes that two bilingual outcomes may exist in Ireland today – subtractive and additive. Subtractive bilingualism occurs when a majority language replaces a minority language (Lambert, 1990). This is similar to when students from Gaeltacht areas, attending primary and secondary schools in that locality, as well as students in Gaelscoileanna and Gaelcholáistí, transfer to learning through the medium of English (majority language). This transfer may be at second or third level education. Prior to this transition point the students would have learnt through the medium of Gaeilge (minority language). It is anticipated that this transition phase would be more difficult for students from Gaeltacht schools as Gaeilge is their home language and the language of the community. For students in Gaelscoileanna and Gaelcholáistí, English is primarily their mother-tongue. In the case of additive bilingualism the students choose to study through the minority language (Gaeilge) even though English-medium education is available to them. This provides a rationale for the establishment of Gaelscoileanna and Gaelcholáistí in Ireland. Positive findings from additive contexts (immersion education) have been reported, in particular from studies carried out in Canada (e.g. Swain, 1996; Turnbull et al, 2000). These studies provide support for the Gaeilge-medium education movement in Ireland. However, both types of bilingual contexts need to be examined, as well as assessing whether a relationship exists between the type of Gaeilge-medium education attended and students' mathematical achievement in English-medium education. This will need to be investigated at both transitions – primary to second level and second to third level education (English-medium).

The review of literature has highlighted the need to look at Gaeilgeoirí's language proficiencies and the influence of this on mathematical learning and understanding. Accordingly this can help examine the type of school attended in order to evaluate if additive and subtractive bilingualism are evident in an Irish context. Cummins' cognitive theories of bilingualism highlight the need for proficiency in both languages in order to cope with and receive cognitive benefits from learning through the medium of a second language.

4.

The Study

4.1 Introduction

The relationship between language and mathematics learning is complex and multifaceted. This investigation examined the influence of bilingualism on mathematics learning at second and third level education in Ireland. The author's principal focus in this study was the language aspect of bilingualism with a secondary focus on cultural and pedagogical influences. The issues under investigation are complex so it was necessary to select a methodological approach that facilitated an in-depth investigation into the Irish context. The theoretical framework employed by the author is unique in that is draws on and combines a number of different areas such as psycho-and socio-linguistics, mathematics registers, and pedagogical and cultural factors, and has made a significant contribution to literature in this area of research. Accordingly, this Chapter provides a description of the study and the theoretical framework, along with the rationale for the selection and implementation of a mixed method approach.

4.2 Description of the study

The study incorporated three phases and each will be discussed in detail while also highlighting their contribution to the overall design of the project (Fig. 4.1, see Ní Ríordáin, 2008). In ***Phase 1*** a comprehensive review of literature was carried out in order to gain an in-depth knowledge of the area of learning mathematics through the medium of a second language, as well as examining areas such as psycholinguistic theories, bilingualism and bilingual education. This Phase incorporated and combined conclusions and recommendations from an undergraduate dissertation with exploratory research. Phase 1 facilitated the design of the methodology and

development of test instruments[†] to be employed in Phase 2 of this research project. ***Phase 2*** was devoted to data collection at both transitions, from primary to second level and from second level to third level education. The analysis of the data collected was carried out in ***Phase 3*** and relevant findings discussed.

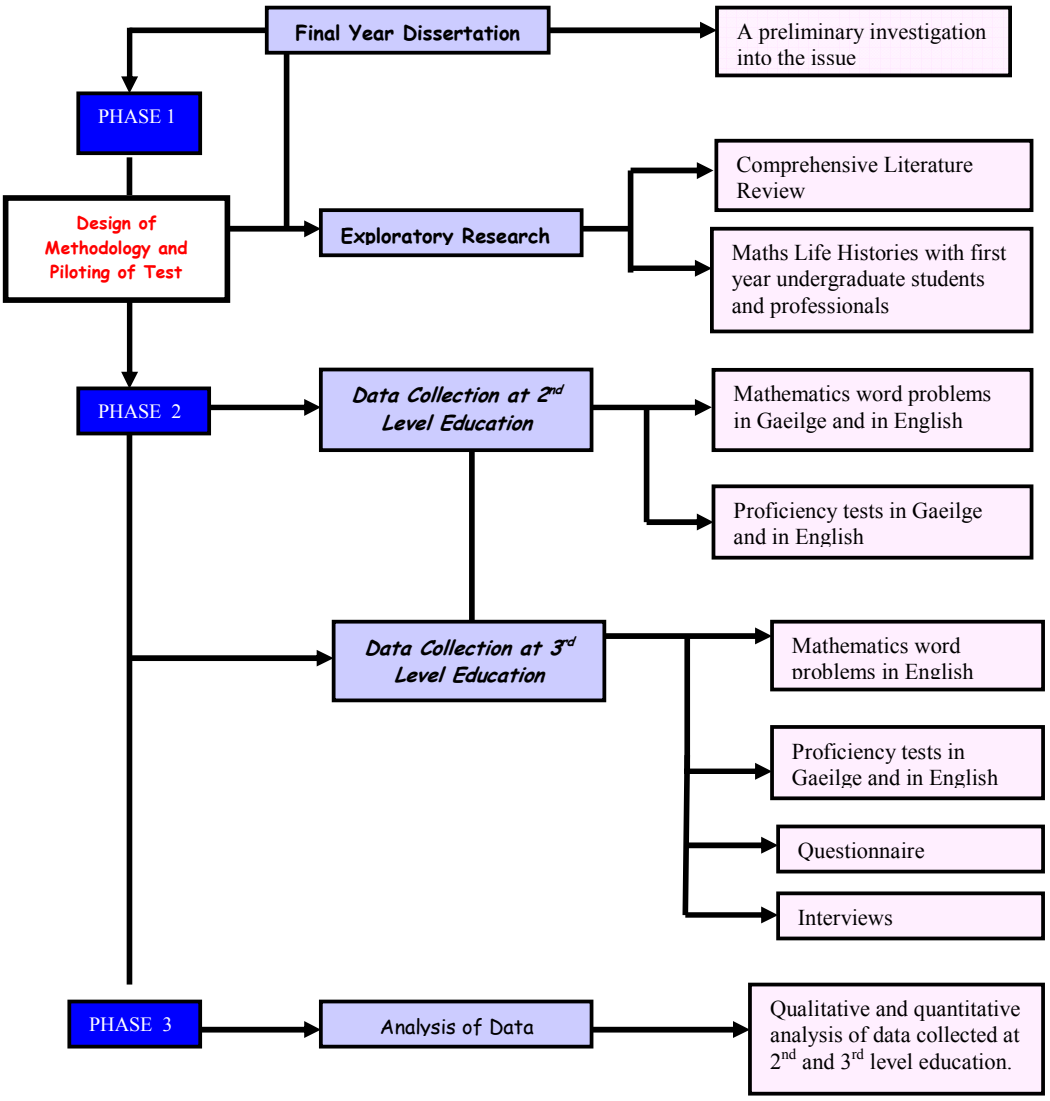


Figure 4.1 An overview of the three-phased research design implemented in the research project.

[†] Please contact the author (mniriordain@stpats.ie) if you would like to obtain a copy of the test instruments.

4.3 Research Questions

This study was motivated by the author's reflections on her own experience of coping with a new language of learning in mathematics. Initially the investigation focused on the effect of changing the language of instruction on mathematics learning (Phase 1) and further exploratory research, using a Maths Life Histories methodologies identified the specific difficulties encountered by Gaeilgeoirí (students who learn through the medium of Gaeilge) when transferring to English-medium education (Phase 1). This is the first study on bilingualism and mathematics education to employ a Maths Life Histories approach, thus a significant contribution to developing methodologies for investigating this area of research. Phase 1 led to more in depth investigations at the primary to second level and second to third level interfaces so as to get a clear understanding of the experiences and challenges faced by Gaeilgeoirí (Phase 2).

Thus the research project evolved and was shaped by previous findings and considerations. The main phase of data collection (Phase 2) was guided by the following **key research questions**:

1. Is performance on mathematics word problems for Gaeilgeoirí through the medium of English affected by their level of language proficiency in English and in Gaeilge? If so, what is the affect and degree of influence that bilingualism has on mathematics learning for Gaeilgeoirí in this study?
2. Is there a significant difference in mathematical learning through English between Gaeilgeoirí transferring from Gaeltacht schools (subtractive bilingualism) and Gaeilgeoirí transferring from Gaeilge-medium schools (additive bilingualism)?
3. Do particular features of the English mathematics register cause difficulty for Gaeilgeoirí at first year second level education and first year undergraduate education?
4. Do cultural and pedagogical factors influence Gaeilgeoirí's transfer from Gaeilge-medium to English-medium mathematics education?

These are the four key research questions that this study addressed in order to identify the potential challenges that Gaeilgeoirí may encounter when transferring to English-medium mathematics education.

4.4 Significant Contribution of the Theoretical Framework

The next section of this study presents a theoretical framework for investigating language issues in mathematics that can be employed in diverse language contexts (the Irish context in this case) and can help interpret the findings emerging from a particular context, hence the significance of this study. It is important to note that the theoretical framework presented here for investigating the transfer between different languages for learning mathematics arose out of the author's doctoral study, no such framework existed prior to this. While investigating the Irish context, the need for such a framework, became apparent to the author and accordingly the in-depth literature review and the research findings emerging from the Irish context influenced the development of the framework presented in this Chapter. The framework is unique in that is draws on and combines a number of different areas such as psycho-and socio-linguistics, mathematics registers, and pedagogical and cultural factors (see Table 4.1). The theoretical framework presented can be employed in order to investigate other bilingual/multilingual learning contexts. Given the increasing number of students learning in a dominant language that is not their first language, these findings are important to mathematics education (Adler, 2001).

4.5 Theoretical Frameworks Employed in this Research Project

Cummins' Threshold Hypothesis (1976) was the primary theoretical framework utilized in this investigation and naturally influences the research design and methodology employed by the author. This Hypothesis recognizes the importance of investigating both languages of learning and their influence on mathematics learning. Given that the Gaeilgeoirí in this study are transferring from learning mathematics through the medium of Gaeilge to learning through the medium of English, this Hypothesis best reflects the situation present in Ireland. Also in Chapters 2 and 3 a

number of language theories, cultural and pedagogical issues, and the mathematics register were discussed. These were employed as theoretical lenses/tools for the analysis and interpretation of the findings emerging from the study. Given the diverse nature of these lenses/tools they highlight the complexity of bilingualism and its influence on mathematics education. However, a number of other key theoretical frameworks influenced the design of the research project and the analysis of the data gathered. These frameworks will be discussed individually and reference will be made to their contribution to the investigation undertaken.

4.5.1 Ellerton (1989) – 'A Framework for Interpreting Language Factors in Mathematics Learning'

This model shows the need to link the various aspects of language factors in mathematics learning and was employed by the author in interpreting the outcomes of the data collected at both transitions in the Irish education system. The framework can be viewed from a 3-dimensional perspective. When assessing it from an overall perspective it can be seen that culture occupies the entire classroom, and that communication within this culture is of key importance. Communication and language become central factors in issues such as socio-linguistics, natural language, psycho-linguistics, problem solving and classroom discourse which intersect with each other, and with most parts of the framework. This model depicts the centrality of the teacher, the mathematics classroom and the curriculum in recognising language issues in mathematics. These were key issues identified by the author at the transition from second to third level education.

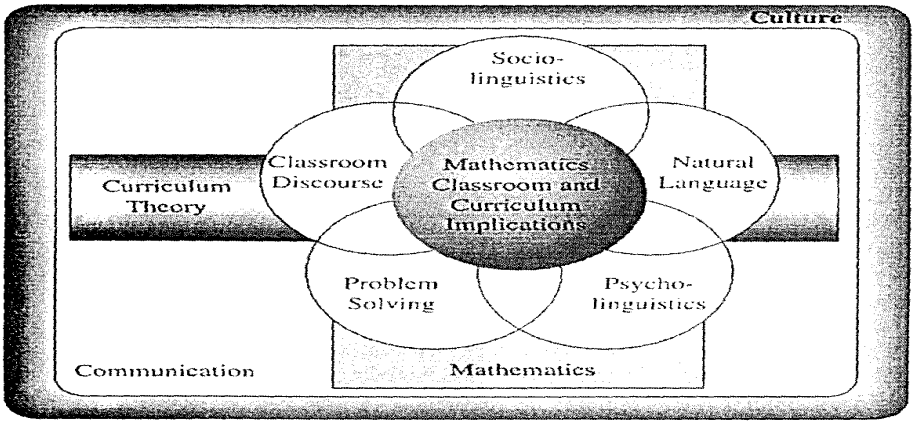


Figure 4.2 A framework for interpreting language factors in mathematics learning (Ellerton, 1989).

4.5.2 Gawned (1990) – 'A Socio-Psycho Linguistic Model'

This framework is based on a model of language learning and summarises a theoretical overview (Ellerton & Clarkson, 1996). The author employs this framework as it best reflects the nature of interaction between natural language and the mathematics register, and how language can influence mathematics learning and understanding.

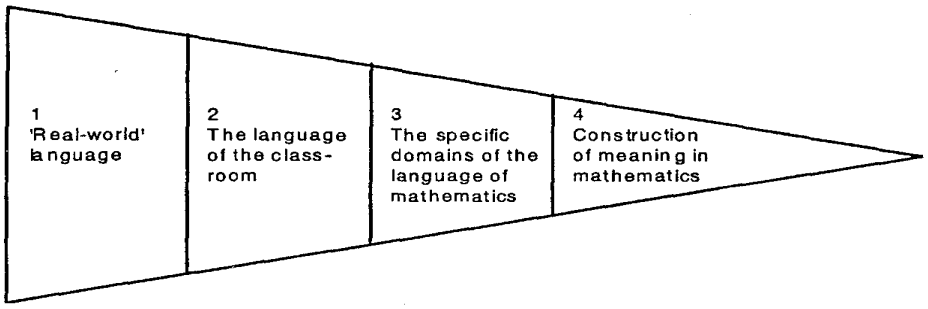


Figure 4.3 A summary of Gawned's socio-psycho-linguistic model

What is of particular importance here is that Gawned (1990) acknowledges that the language of the classroom has a very important influence on students' understanding of mathematics, and that each classroom has a unique culture of its own. Gawned (1990) also discusses the discourse patterns found in mathematics classrooms. They tend to be dominated by rules, function within strict relationships and are teacher centred. Thus, this framework reflects the nature of mathematics classrooms and how language plays a key role in learning, particularly the language of the teacher and the textbook, while also highlighting the cultural influences on mathematics education. This framework facilitated the author's interpretation of and illustrated the relationships between key findings, in particular the findings emerging from the interviews conducted at the transition to third level education.

Both Ellerton's (1989) and Gawned's (1990) frameworks provided a theoretical structure for the design methodology and analysis of data collected in relation to bilingualism and mathematics education in Ireland. They establish the need to link the various aspects of language factors in mathematics learning such as socio-

linguistics, natural language, psycho-linguistics, problem solving and classroom discourse which intersect with each other. The frameworks provide a rationale for the author's study as they demonstrate how language and the language of instruction are key areas in the learning and teaching of mathematics. Thus, this has implications for students learning mathematics through the medium of a second language.

4.5.3 Newman Research Method (1977)

Newman's (1977) research is very focused on written aspects of mathematics and its link with language. Her framework for attending to mathematical tasks is of great significance considering that learners are faced with it continuously when dealing with textbooks and exams. She states that when confronted with a written mathematical word problem a person needs to go through a fixed sequence of events when answering the problem:

- Reading (or decoding).
- Comprehension.
- Transformation (or Mathematising).
- Process skills.
- Encoding.

Newman also assigned a category, 'Careless', to errors resulting from unknown factors. Additionally, the Newman procedure involves an interview with subjects, after attempting to solve the word problem, so as to determine where they had difficulty answering it. They are asked a sequence of questions with the intention of establishing whether they can read the question; comprehend what they have read; conduct an appropriate mental transformation from the words of the problem to the choosing of a mathematical strategy; employ the process skills required by the selected strategy; and encode the answer in an appropriate written form. This framework provided justification for employing mathematics word problems as a means of assessing mathematics performance, while addressing potential language issues Gaeilgeoirí may encounter when confronted with mathematics through the medium of English.

Table 4.1 Research Questions, Theoretical Frameworks and Theoretical Lenses/Intellectual Tools employed in the Study

Research Question	Key Theoretical Framework (TF)	Influence of TF on Research Design	Influence of TF on Analysis	Significant Lens/ Tool Employed
1	**Cummins' (1976).**	Language proficiency tests.	▪ Investigated if there is a relationship between mathematics performance and language proficiency.	○ Cummins' Developmental Interdependence Hypothesis (1979).
	Newman (1977).	Mathematics word problem tests. Language use survey. Interviews.	▪ Investigated where Gaeilgeoirí encountered difficulties with mathematics through the medium of English and the language use they employed in mathematical problem solving.	○ BICS vs CALP. ○ Psycholinguistic theories.
2	**Cummins' (1976).**	Language proficiency tests.	▪ Investigated the language proficiencies of Gaeilgeoirí in Gaeltacht and Immersion schools and their performance in mathematics.	○ Second language acquisition. ○ Cummins' Developmental Interdependence Hypothesis.
	Ellerton (1989).	Psycho & Socio – linguistics permits the distinction between Gaeltacht schools and immersion schools.		
3	**Newman (1977)**	Mathematics word problem tests. Interviews.	▪ Assessed the mathematics word problems they experience most difficulty with and the potential sources of difficulty within these problems.	○ The mathematics register. ○ Language features that impede learning.
	Gawned (1990)	Identification of potential sources of difficulty associated with mathematics word problems in relation to natural language and mathematics register.	▪ Investigated where Gaeilgeoirís' understanding breaks down when engaged in mathematical problem solving.	○ Sapir (1949)-Whorf (1956) Hypothesis. ○ Newman Research Method (1977).
4	**Ellerton (1989)**	Questionnaire. Interviews.	▪ Examined pedagogical and cultural influences on Gaeilgeoirís' conceptions of mathematics and how they influence the transition to English-medium education.	○ Understanding & culture. ○ Cultural issues. ○ Types of mathematical understanding. ○ Language and understanding. ○ Pedagogical issues.
	Gawned (1990)	Questionnaire. Interviews.		

4.6 The Relationship between the Research Questions and Theoretical Frameworks

The relationship between the research questions, the theoretical frameworks supporting the investigation of these questions, and the significant analytical lenses/tools employed in the analysis of the data and the interpretation of findings is demonstrated in Table 4.1. These theoretical lenses and intellectual tools were presented and discussed in Chapters 2 and 3.

4.7 Research Design

When designing a plan of research a number of components need to be taken into consideration. These include the elements of inquiry, the approaches to research and the design process of the research (Creswell, 2003). Knowledge claims, strategies and methods (i.e. elements of inquiry) unite to produce different methods in research. These in turn are converted into processes in the plan of research (Creswell, 2003). Thus, when devising a research project it is necessary to assess the knowledge claims involved in the study, to take into consideration the strategies of inquiry to be implemented, and to establish specific methods. Resulting from this the researcher can choose to implement a quantitative, qualitative or mixed method approach to their research project (Creswell, 2003).

Fundamental to the selection of a research method are the research questions being addressed and investigated. Due to the nature and complexity of issues being investigated, this study employed a mixed-methods research approach. The combination of the qualitative and quantitative research paradigms evident in mixed-methods is one of its primary characteristics and often contributes to better-quality research in comparison to single method research (Johnson & Onwuegbuzie, 2004). The quantitative and qualitative paradigms are distinguished primarily on their ontology (assumptions concerning reality), epistemology (knowledge of that reality) and methodology (the particular ways of knowing that reality). As a consequence, two dominant research cultures have emerged: one culture a strong advocate of the use of profound, rich observational data, the other supporting the superiority of concrete, generalizable data. Although the use of mixed-methods has stimulated much debate,

its increase in attractiveness can be related to its ability to enlarge the extent of a study and advance its methodological supremacy (Greene & Caracelli, 1997). Quantitative and qualitative methods are simply tools; combining them facilitates the resolution of significant research questions.

For this research project a mixed-methods approach is evident in, and justified by, the use of methodological *triangulation* (Denzin, 1970). Denzin (1970) discriminates between two sub-types of methodological triangulation. These are the within-method approach and between/across-method approach. The within-method consists of utilizing the same method on separate occasions or employing a variety of techniques within a given method. On the other hand, the between-methods approach consists of the genuine mixing of methods within a single research project, as is evident in this research project. By employing methodological triangulation, this in turn facilitates the investigation of *different research questions*, the methods complement one another in that they provide an *explanation* of findings generated and the implementation of a mixed methods approach engenders *completeness* to the research project.

4.8 Project Design – A Three-Phased Approach

The primary aim of this research project was to investigate Gaeilgeoirís' transition from learning mathematics through Gaeilge to learning mathematics through English. The author set out to investigate if language impacts positively or negatively on mathematics learning and teaching for Gaeilgeoirí. The investigation was divided into three phases, each of which will be discussed.

4.8.1 Phase 1: Exploratory Research

This study originated when the author undertook a final year dissertation in this area of research as part of an undergraduate degree (Ní Ríordáin, 2005). It became apparent that no research had been carried out in Ireland prior to this project (MacNamara's (1966) work was concerned with students from English backgrounds learning through the medium of Gaeilge). Thus there was no available data to work with so after an extensive review of literature had been carried out two pilot studies were undertaken; one at second level and one at third level education. This was the

first phase of the research project and further literature review and exploratory research utilising a Maths Life Histories approach (Coben & Thumpston, 1994) was carried out during the academic year 2005/2006. The primary aim of this first phase was to investigate the extent to which performance in mathematics could be attributed to the language of instruction, to examine the experiences of Gaeilgeoirí in the transition process and to establish and clarify the key issues facing Gaeilgeoirí in the transition from learning mathematics through Gaeilge to learning mathematics through English. This was to ensure that subsequent research carried out by the author addressed the relevant issues and contributed to development in this research domain. The conclusions and recommendations inferred from Phase 1 of this research project contributed to the formulation of four key research questions (Phase 2). Resulting from this, test instruments were compiled and piloted for both transitions at second and third level education consisting of mathematics word problems, proficiency tests in English and in Gaeilge, a questionnaire and interview schedule. The findings of Phase 1 are reported and discussed in Chapter 5 of this monograph. Table 4.2 provides a summary of Phase 1 of this research project.

Table 4.2 Summary of Phase 1 of Research Project.

PHASE 1		
CHARACTERISTICS	**PRIMARY – SECOND LEVEL**	**SECOND – THIRD LEVEL**
KEY THEORETICAL FRAMEWORKS	• Cummins (1976) • Newman (1977)	• Ellerton (1989) • Gawned (1990)
RESEARCH AIMS	• Investigate the extent to which performance in mathematics could be attributed to the language of instruction.	• Examine the experiences of Gaeilgeoirí in the transition to English - medium mathematics education. • Establish and clarify the key issues facing Gaeilgeoirí in the transition from Gaeilge-medium to English-medium mathematics education.
RESEARCH METHOD	*Quantitative* • Mathematics word problems in English and in Gaeilge. These word problems were designed using Heller & Greeno Classification (1978).	*Qualitative* • Questionnaire. • Maths Life Histories.
ANALYSIS	• Quantitative – Additive & Subtractive responses (Jones, 1982).	• Qualitative – comparison of responses and NVivo for assisting in analysing the Maths Life Histories data.

THEORETICAL LENSES/ INTELLECTUAL TOOLS	• Cummins (1976) • Cummins (1979) • Newman (1977)	• Mathematics register. • Sapir (1949)–Whorf (1956). • Cultural issues – Bishop (1988) Sierpinska (1994). • Pedagogical issues - Bernstein (1971) Nickson (1992) Hiebert (1997) • Understanding & culture/ language – Sierpinska (1994).

4.8.2 Phase 2: Investigation at Key Transition Stages

This Phase of the research was concerned with in-depth data collection at both the transition from primary to second level education and from second to third level education (see Table 4.3). Students were sourced from Gaeltacht schools and from Gaelscoileanna/Gaelcholáistí so as to facilitate the comparison between additive and subtractive bilingual learning environments. Data collection at both transitions utilized a control group of students who were from a similar educational background but had learnt mathematics entirely through the medium of English at primary and second level education. This is to ensure the validity and reliability of test instruments employed and accordingly the findings emerging from the data collected, as well as allowing for the comparison between bilingual with monolingual mathematics students in Ireland. This phase of research can be divided into two sections – the primary to second level transition and the second to third level transition.

Table 4.3 Summary of Phase 2 of the Research Project.

PHASE 2		
CHARACTERISTICS	**PRIMARY – SECOND LEVEL**	**SECOND – THIRD LEVEL**
KEY THEORETICAL FRAMEWORKS	• Cummins (1976) • Newman (1977)	• Cummins (1976) • Newman (1977) • Ellerton (1989) • Gawned (1990)
RESEARCH QUESTIONS	• **1.** Is performance on mathematics word problems for Gaeilgeoirí through the medium of English affected by their level of language proficiency in English and in Gaeilge? If so, what is the affect and degree of influence that bilingualism has on mathematics learning for Gaeilgeoirí in this study? • **2.** Is there a significant difference in mathematical learning through English between Gaeilgeoirí transferring from Gaeltacht schools (subtractive	• **1.** Is performance on mathematics word problems for Gaeilgeoirí through the medium of English affected by their level of language competency in English and in Gaeilge? If so, what is the affect and degree of influence that bilingualism has on mathematics learning for Gaeilgeoirí in this study? • **2.** Is there a significant difference in mathematical learning through English between Gaeilgeoirí transferring from Gaeltacht schools (subtractive bilingualism) and Gaeilgeoirí transferring from Gaeilge-medium schools (additive bilingualism)?

	bilingualism) and Gaeilgeoirí transferring from Gaeilge-medium schools (additive bilingualism)? • **3.** Do particular features of the English mathematics register cause difficulty for Gaeilgeoirí at first year second level education and first year undergraduate education?	• **3.** Do particular features of the English mathematics register cause difficulty for Gaeilgeoirí at first year second level education and first year undergraduate education? • **4.** Do cultural and pedagogical factors influence Gaeilgeoirí's transfer from Gaeilge-medium to English-medium education?
RESEARCH METHOD	*Quantitative* • Mathematics word problems in English and in Gaeilge. These word problems were designed using standard mathematics textbooks at 2nd level education in Ireland. • Language proficiency tests in Gaeilge and in English.	*Quantitative & Qualitative* • Mathematics word problems in English and in Gaeilge. These word problems were designed using the PISA (2006) framework and Hater & Kane (1975). • Language proficiency tests in Gaeilge and in English. • Questionnaire. • Interviews.
ANALYSIS	• Quantitative – SPSS (Version 15).	• Qualitative – comparison of responses and NVivo. • Quantitative – SPSS (Version 15).
THEORETICAL LENSES/ INTELLECTUAL TOOLS	• Cummins (1976) • Cummins (1979) • Newman (1977) • Mathematics register.	• Cummins (1976). • Cummins (1979) • Newman (1977) • Mathematics register. • Sapir (1949)–Whorf (1956). • Cultural issues – Bishop (1988) Sierpinska (1994). • Pedagogical issues -Bernstein (1971) Nickson (1992) Hiebert (1997) • Understanding & culture/ language – Sierpinska (1994).

4.8.3 Phase 3: Analysis and Contribution of Research

Quantitative and qualitative analysis was undertaken on the data collected. This Phase was important as the author derived key findings and significant insights into the area of bilingualism and mathematics education in Ireland and as well as contributing to international literature in this area of research. Accordingly conclusions and recommendations from the investigation are indicated.

4.9 Validity and Reliability

Validity and reliability are of utmost importance when designing and implementing a research project. In particular, the validity and the reliability of research methods employed will determine the significance of the data collected and analysed, and thus

influence the significance of the conclusions obtained from the data (Leedy & Ormrod, 2001).

4.9.1 Validity

The validity of a test instrument can be defined as "the extent to which the instrument measures what it is suppose to measure." (Leedy & Ormrod, 2001, p.31). Validity is strengthened through the selection of a suitable research methodology, and furthered by the selection of appropriate research tools. In this study the author employed a mixed-methods approach and a variety of instruments/tools in order to gain an insight into the issue of learning mathematics in a second language for Gaeilgeoirí. Given that this is the first study of this type of research to be undertaken in Ireland, the author employed research tools consistent with those employed in similar studies internationally (e.g. Barton, 2005; Clarkson, 2007). In the context of this study, the validity of the mathematics word problem test instruments employed at second and third level education was ensured through appropriate piloting with groups of students at each level and feedback was obtained from teachers/lecturers at each particular level of education. The proficiency tests employed were standard tests appropriate for each age group (Cambridge English Language Proficiency Tests and UL Aonad na Gaeilge Language Proficiency Tests) and thus had undergone rigorous testing prior to standardisation ensuring their validity as measures of language proficiency. Likewise, the questionnaire utilized at third level underwent appropriate piloting with students of a similar background to those involved in the final data collection phase. The samples chosen were representative of the population of Gaeilgeoirí (students learning through the medium of Gaeilge) i.e. both studies incorporated students transferring from Gaeltacht schools (Maintenance Heritage Language Schools) and from Gaelscoileanna/Gaelcholáistí (Immersion Schools). At both levels of education control groups of monolingual English students were sourced and involved in the studies, thus facilitating the validation of research findings and conclusions derived from the data. Several types of validity exist in relation to research undertaken and those that are of importance to this study are:

4.9.2 Reliability

Reliability can be defined as "the consistency with which a measuring instrument yields a certain result when the entity being measured hasn't changed." (Leedy & Ormrod, 2001, p.31). This implies that there should be a high level of correlation if

one were to perform the same type of research employing the same test instruments but with a different sample of participants. Within this research project, the author sought reliability through the implementation of a mixed-methods approach; all participants completed the same research instrument within each element of the research; a rigorous approach to data collection, analysis and write up was adopted by the author; and by triangulation i.e. data was collected using a variety of sources.

4.10 Ethics

Ethical approval was sought through the University of Limerick Ethics Committee. Ethical issues were recognised and guidelines were adhered to and thus the research methodology was designed such that:

- Participation in the study was strictly voluntary.
- Participants had the right to withdraw at any time while the research was being conducted.
- Parental/Guardian approval was sought for participants under the age of eighteen.
- All participants were given a clearly worded information sheet on the purpose of the research being undertaken and what would be required of them should they agree to participate.
- All participants signed a consent form before participating in the research.
- Participant confidentiality was ensured through the allocation of an individual code number and pseudonym, which was used in all documentation.
- The data is used only for research purposes.
- The data is stored according to UL Ethic's regulations.

4.11 Researcher Distance

The idea of this research came about from the author's own experience of having to go through a similar transition on entering third level education. Thus she is aware that she came to this research with her own biases, assumptions and expectations or research outcomes. However, researcher distance and objectivity was established by firstly acknowledging her biases and predispositions, by employing a mixed-methods approach and variety in data collection tools, and by applying professional standards

in the carrying out and the analysis of qualitative and quantitative data. This in turn provides confidence in the researcher's findings. *"In any form of research the significance of the data must always be judged relative to the researcher's explicit or implicit theories and assumptions."* (Goos & Galbraith, 1996, p.234). For this research project, these assumptions have been acknowledged, at least to some degree.

4.12 Conclusion

This Chapter has provided an account of the theoretical framework employed in this research project, as well as the necessary theoretical and practical considerations needed to be taken into account in the implementation of this research project. A mixed-methods approach was adopted and a three-phased approach was implemented in order to address the research questions. Chapters 5 and 6 will present and discuss the key findings emerging from this research study in the Irish context.

5.

Bilingualism and Mathematics Education in Ireland – Study Findings.

5.1 Introduction

This Chapter will report on the findings from Phases 1, 2 and 3 of the research project. Phase 1 of the research project was concerned with identifying some of the key issues facing Gaeilgeoirí in the transition to English-medium mathematics education and this Phase of investigation influenced the development of the key research questions to be addressed during Phase 2 of this research project. Consequently Phase 2 (data collection at both transitions) and Phase 3 (analysis and discussion of data collected) provides a more comprehensive and relevant enquiry into the area of bilingualism and mathematics education in Ireland.

5.2 Phase 1 of the Research Project

Phase 1 incorporated and combined conclusions and recommendations from an undergraduate dissertation with further exploratory research. The undergraduate work (March, 2005) incorporated the use of mathematics word problems at second level (Heller & Greeno Classification, 1978) with parallel versions in English and Gaeilge, whereas a questionnaire was utilised at third level to examine the experiences of Gaeilgeoirí in the transition to English-medium mathematics education. The findings of this work are revisited in this section with a view to establishing the key issues to be pursued in the follow-up exploratory research. This follow-up exploratory work incorporated the use of a Maths Life Histories approach (Coben & Thumpston, 1994). This section will firstly present the findings from revisiting the undergraduate dissertation, followed by the findings from the exploratory research and finally the recommendations for Phase 2 of the research project.

5.2.1 Key Findings from the Undergraduate Study

Gaeilgeoirí (4 in total) who had transferred from Gaeilge-medium primary education to English-medium second level education completed a mathematics word problem test in English and a parallel version of the test through Gaeilge. The Heller and Greeno (1978) classification of problems was employed in this study. It states that any arithmetic word problem, which can be solved by the use of one procedure of addition or subtraction must belong to one of three categories – Change, Combine or Compare (Heller & Greeno, 1978). The 3rd level undergraduate students who participated (Gaeilgeoirí in the transition from Gaeilge-medium education, 5 in total) completed a questionnaire that was sent online.

At the primary to second level transition, the author found that the subjects performed better when the word problems were presented in Gaeilge as opposed to English, in particular on the more difficult word problems (Change 3, 5, & 6; Combine 2; Compare 4, 5, & 6). The subjects never under-performed when the questions were presented in Gaeilge, but for Change 2, Change 4, Combine 1, Compare 2 and Compare 3 all the subjects got the questions correct in both languages. It is clear that the language of presentation did not affect their performance on these questions. However, this may be attributed to them being classified as the easier type of word problems (Lean et al, 1990).It can be deduced that the overall poorer performance of the subjects on the problems presented in English was not simply some innate lack of intelligence as they proved themselves to be smart and alert when answering the same problems in their mother-tongue (Gaeilge). The author suggests that it was due to the difficulties they experienced with the language of presentation (English), which may have contributed to misunderstanding in problem transformation, and thus led to the use of inappropriate process skills. As a consequence, the subjects were experiencing difficulties with their mathematics due to the language of instruction and presentation and this will have a negative impact on their learning.

Overall, the respondents found studying through the medium of English difficult, with two of the participants quoting it as being “extremely difficult”. The main problems they encountered were in understanding the lecturers/tutors, as well as comprehending the content of the modules. This in particular was in relation to the mathematical terminology in use in these modules, which led to confusion and misunderstanding.

Overall, the respondents felt that learning through the medium of English and the difficulties they were experiencing was contributing to a negative educational experience at third level. All respondents commented on having to spend extra time on mathematics outside of lecture/tutorial time with the least amount noted being 3 hours per week. Four of the five respondents still thought and performed mechanical operations through Gaeilge. They felt that this was the most natural way for them. Three of them acknowledged that they resolved to trying to translate the content in an attempt to understand the material, but they admitted that this led to confusion and incoherence of the material. None of the respondents had approached any member of staff about the problems they were experiencing and none commented further on this. Clearly, these students were experiencing difficulty in studying mathematics through the medium of English. Specific types of problems that can be expected include difficulty in understanding technical terminology and vocabulary, context, phrases and methods of arguing, the use of everyday words (with mathematical meaning) and relying on translation to help in understanding the content. There is an obvious need for support services to be introduced to cater for these students.

5.2.2 Key Findings from the Exploratory Research

A qualitative approach was employed in this exploratory phase of the research project so as to gain a more in-depth understanding of the experiences of Gaeilgeoirí in the transition to English-medium mathematics education (Leedy & Ormrod, 2001). More specifically a Maths Life Histories approach was utilized which involved the use of semi-structured interviews which were used to gather narrative accounts of the subjects' experiences (Coben & Thumpston, 1994). Interviews were conducted with first year mathematics students at third level, and with professionals who have experienced the transition during their education/training. In total six subjects were interviewed (3 students & 3 professionals) and all subjects were required to have transferred from learning mathematics through Gaeilge at second level to learning mathematics through English at third level.

From the analysis of the Maths Life Histories a number of relationships were established (Figure 5.1). These relationships are shown diagrammatically and are discussed in pairs in the following paragraphs.

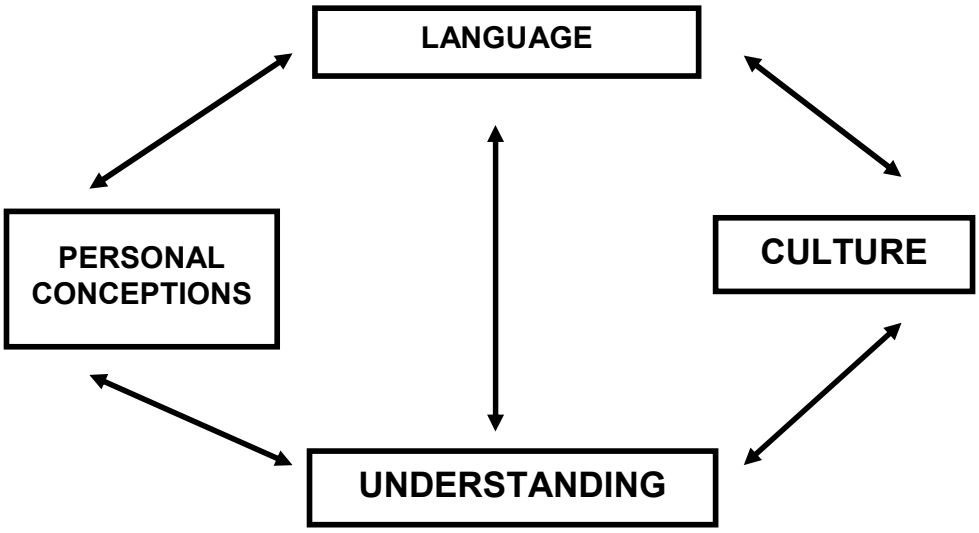


Figure 5.1 The relationship between research findings.

Language – Personal Conceptions:

Mathematics terminology was the primary source of difficulty when transferring from learning through Gaeilge to learning through English for all subjects interviewed. In particular, it was the "basic mathematics" or fundamentals that they had learnt through Gaeilge at primary/secondary level. These "basic mathematics" included simple operations such as addition (suimiú), subtraction (dealú); types of numbers e.g. integer (uimhir), complex (casta); and labels e.g. denominator (ainmneoir), hypotenuse (taobhagán).

However, given the large emphasis that *all* the subjects placed on mathematics terminology it was surprising that the majority of them did not perceive language as a component of mathematics. Perceptions of mathematics ranged from "just solving stuff", to "loads of formulas for solving problems" to "figures and computations and working out answers". However, the interviewees that had pursued mathematics beyond degree level had a more developed understanding of mathematics and could see its relationship with everyday life. But yet their personal conceptions of mathematics failed to develop an association between mathematics and language.

Language – Culture:

English-medium education is largely the norm in Irish third level institutions thus catering for the majority and isolating the minority. The transition from Gaeilge-

medium second level education to English-medium third level education was described by the interviewees as "difficult", "hilarious!", "hell" and "being thrown in the deep end". Many subjects expressed the difficulty and discomfort experienced on entering third level education due to language and cultural differences. But it was not just language that caused a barrier. A number of other issues became prominent such as the culture of third level institutions, which are directed towards independent and self-directed learning. None of the subjects interviewed had approached their lecturers/tutors about difficulties they were experiencing due to the language barrier and few were aware of support services available to them. The general consensus was a fear of being perceived as "different (because of their language background)", "looking stupid because you don't know the mathematics" and a fear of lack of understanding of the issue on behalf of those providing support. All relied heavily on the support of *close* friends whom they felt comfortable talking to about their mathematics problems.

Language – Understanding:

Discussion of these types of understanding, as well as the relationship between language and understanding became an important aspect of the mathematical life histories. It soon became apparent that the interviewees associated learning/"knowing" their mathematics through Gaeilge with understanding and comprehending their mathematics. Having English as the medium of instruction influenced their learning strategies. All subjects acknowledged that they "learnt stuff off" for examinations and that they did not understand some of the concepts behind the mathematics that they were learning. They described learning the English mathematics register as "relearning the words", not as developing mathematical understandings through English and transferring skills learnt through Gaeilge to English. A considerable effort was dedicated towards rote learning and practice of mathematical questions. Thus the subjects relied on procedural knowledge with little development of relational understanding.

Personal Conceptions – Understanding:

The subjects' experiences in mathematics at all levels of education were intermingled with their perceptions of their "ability" and understanding of mathematics. Three of the subjects had positive experiences at primary and secondary level and thus opted to

study degree courses with mathematics as a large component. They felt confident in completing mathematical tasks and felt that the transition to English-medium education did not hinder their progress in mathematics long term. The remaining three had negative experiences, particularly at second level, and were pursuing degree courses where mathematics was only required at service level. Moreover they felt that they lacked ability in mathematics and this in turn was a source of negative attitudes towards the topic.

Culture – Understanding:

The majority of the participants spoke of the new environment they were suddenly immersed in. Subjects spoke of the expectation of having a certain competency in mathematics before entering university, but that this was hindered by the level of mathematics taken at second level and also by the language barrier. Abandonment was a significant issue for three of the subjects in particular. Progressing from having been "spoon-fed" all through second level, as well as having the constant support of teachers, to suddenly being an insignificant number in a crowd was a difficult transition for them. This is consistent with universities' strategy of producing independent learners. All of the interviewees relied on procedural knowledge in order to succeed in examinations, and little time was given to the development of conceptual understanding.

5.2.3 Summary of Key Findings from Phase 1

The studies were carried out on a small scale and thus are not generalizable but recommendations are drawn up for further investigation in this research project (Phase 2). The author believes that these are the most prevalent issues confronted by Gaeilgeoirí experiencing the transition and thus merit further investigation in order to identify the key challenges they encounter at each transition.

The recommendations for Phase 2 include:

- If important progress is to be made in improving mathematics education for those learning mathematics in a second language (English), further investigation is required in relation to the particular aspects of the English mathematics register which hinder their learning of mathematics and cause difficulties for the students learning in a second language.

- Cummins (1979) conjectured that there may be a threshold level of language proficiency that bilingual children must achieve both in order to avoid cognitive deficits and to allow the potential benefits of being bilingual to come to the fore. This hypothesis should be investigated in relation to Gaeilgeoirí in order to assess whether the level of language competence in both languages (Gaeilge & English) has detrimental/beneficial influences on their learning.
- Ireland possesses both Maintenance Heritage Language and Immersion education (Baker & Prys-Jones, 1998) and both are firmly established within a very small country and thus provides the opportunity for diverse areas of investigation. The literature identifies Subtractive and Additive bilingualism associated with these bilingual educational contexts respectively. There is a need to investigate these two diverse mediums of Gaeilge-medium education in order to assess their significance in contributing to the development of additive and subtractive bilingualism for Gaeilgeoirí.
- The various cultural influences on learning, on understanding and on mathematics need to be examined and assessed in relation to their impact upon the key transition stages – particularly at the second to third level education.

5.2.4 Additional Comment

A key outcome of the preliminary research is the author's four-way conceptualisation of the difficulties encountered by some Gaeilgeoirí in the transition to English-medium mathematics education and the relationships between these difficulties (Figure 5.1). This model is part of the author's contribution to the area of bilingualism and mathematics education as the model forms an important analytical tool for Phase 2 of the research project, derived from the author's own insights into the area of bilingualism and mathematics education in Ireland. This model underpins all research questions being addressed in Phase 2. The model demonstrates the need to look at the relationship between language proficiency and mathematics understanding (Research Question 1, see Section 4.3); the need to investigate the influence of culture and language on mathematics understanding (Research Question 2, see Section 4.3); the need to look at the influence of personal conceptions and language on mathematics understanding (Research Question 3, see Section 4.3); and finally to investigate the relationship between personal conceptions, culture and language on mathematics

understanding (Research Question 4, see Section 4.3). Given the diversity of research questions being addressed it was necessary to employ a mixed methods approach.

5.3 The Main Study (Phase 2 and 3)

The next sections of this chapter will report on the key findings and insights from the main study undertaken on bilingualism and mathematics education in Ireland. Data collection took place at both key transitions in the Irish education system – Primary to Second Level and Second to Third Level educations. A mixed-method approach was employed in the data collection and analysis. The remainder of the Chapter is divided into four parts - each concerned with addressing one of the key research questions of the project.

5.4 Subjects involved in the Main study

The bilingual participants at **second level** were chosen using the following criteria:

- They were required to have studied mathematics entirely through the medium of Gaeilge at primary level,
- That they were currently studying mathematics through the medium of English at second level,
- All subjects were in their first year of second level education.

Subjects from Gaeltacht schools (16 in total) and Gaelscoileanna (21 in total) were used in the study, as well as a control group consisting of monolingual English-speaking students (49 in total). This monolingual group was sourced at the second level schools that the bilingual students were attending and were in the same classes as these Gaeilgeoirí.

At the transition from second to third level education, the bilingual subjects were currently enrolled as **third level** students and were selected if:

- They had studied mathematics entirely through the medium of Gaeilge at primary and at second level education,
- They were now studying mathematics through the medium of English at third level,

- They were in their first year of third level education.

Once again, subjects from Gaeltacht schools (9) and Gaelcholáistí (6) participated in the study, as well as a monolingual control group consisting of six students who had learnt mathematics entirely through the medium of English at primary and second level education and who were currently enrolled in third level education. The students selected were from universities, institutes of technology and colleges of education. Mathematics was a minor part of their degree courses for all students involved in the study.

Table 5.1 Description of participants at each transition in the investigation.

	Bilingual Group	***Monolingual Group (English Control Group)***	***Total Cohort***
Primary – Second Level (Transition 1)	Entire Group *(BG/T1)*: n = 37 Gaelscoil *(BGc/T1)*: n = 21 Gaeltacht *(BGt/T1)*: n = 16	n = 49 *(M/T1)*	n = 86 *(T /T1)*
Second – Third Level (Transition 2)	Entire Group *(BG/T2)*: n = 15 Gaelcholáiste *(BG/T2)*: n = 6 Gaeltacht *(BGt/T2)*: n = 9	n = 6 *(M/T2)*	n = 21 *(T/T2)*

5.5 Key findings in relation to Research Question 1

Research question to be addressed:

1. ***Is performance on mathematics word problems for Gaeilgeoirí through the medium of English affected by their level of language proficiency in English and in Gaeilge? If so, what is the effect and degree of influence that bilingualism has on mathematics learning for Gaeilgeoirí in this study?***

5.5.1 Methodology Employed

At both transitions the bilingual participants completed mathematics word problems in English and language proficiency tests in English and in Gaeilge. Bilingual participants at the transition to English-medium second level education also completed mathematics word problems in Gaeilge. The monolingual groups at each transition completed the same mathematics word problems in English and the language competency test in English so as to facilitate a comparison and act as a control group.

5.5.2 Analysis

All quantitative data was coded and imported into SPSS (Version 15) for analysis. The relevant variables in each of the data sets were explored and tested for normality before applying appropriate statistical tests and assessing the significance of the results. Pearson's Correlation was employed for assessing the relationships between the two languages and performance on the mathematics word problem tests. The correlations are reported along with the significance of each correlation. For assessing the significance in performance between various groups and variables Mann-Whitney U was employed as these variables were skewed (see the appropriate box plots) and significance was set at 0.05 or less.

5.5.3 Language Proficiency Groups

A technique devised by Clarkson (2007) was used to segregate the participants into language proficiency groups. In accordance with their score on the language proficiency test in English, the participants were selected as having comparatively high, middle or low proficiency in English. By rank ordering the scores obtained by the monolingual English control groups, the two scores that divided each group into thirds were recorded and then applied to the bilingual groups, resulting in three sub-groups at each transition. The median score for the proficiency test in Gaeilge was used in order to divide Gaeilgeoirí into comparatively high or low proficiency groups in Gaeilge, at each transition (Clarkson, 2007).

Table 5.2 Threshold scores for the construction of the language proficiency groups.

	English Language Rank Scores	***Gaeilge Median Scores***
Primary – Second Level Transition	High: ≥ 18 Medium: 14-17 Low: ≤ 13 (Out of 25)	Median = 18 High: ≥ 18 Low: < 18 (Out of 30)
Second – Third Level Transition	High: ≥ 8 Medium: 5-7 Low: ≤ 4 (Out of 16)	Median = 51 High: ≥51 Low: < 51 (Out of 65)

Students were then categorised as relatively high proficiency in both languages; dominance in one language (combination of high/low); or relatively low proficiency

in both languages (combination of low/low). Each student was assigned to *only* one of these language proficiency groups. Six of the students dropped out of the analysis at the primary-second level interface because they did not fit clearly within the sub categories due to having a combination of high/medium or low/medium proficiency in the languages. At third level two of the students were not included in the analysis as once again they did not fit clearly within the designated categories (Table 5.3).

Table 5.3 Description of the language proficiency groups.

	Categorisation	***Primary-Second Level***	***Second-Third Level***
High/High	High Gaeilge & High English	n = 14	n = 3
Low/Low	Low Gaeilge & Low English	n = 7	n = 3
Dominant Gaeilge	High Gaeilge & Low English	n = 10	n = 4
Dominant English	Low Gaeilge & High English	n = 0	n = 3
Monolingual	All-English Schooling	n = 49	n = 6

5.5.4 Findings

The analysis of the data collected demonstrates that Gaeilgeoirí's performance on mathematical word problems is related to their linguistic proficiencies in both languages. The author acknowledges that other factors such as parental background, language spoken in the home, social class and IQ may influence Gaeilgeoirí's performance on the mathematical word problems but the primary focus of this aspect of the investigation was to establish direct relationships between language proficiency and mathematics performance on word problems.

Overall, the study found that performance on mathematical word problems in English is related to language proficiency in English for Gaeilgeoirí and for monolingual English speaking students at both transitions, with greater importance at the transition to third level education (Table 5.4).

Table 5.4 Correlations between mathematics performance (in English) and English language proficiency.

	Groups & Pearson's Correlation	*Significance*	*Description*
Primary – Second Level	[3] T-T1: r = 0.48 M-T1: r = 0.52 BG-T1: r = 0.41	p < 0.01 p < 0.01 p < 0.05	All are moderate correlations but are significant
Second –Third Level	T-T2: r = 0.69 M-T2: r = 0.91 BG-T2: r = 0.65	p < 0.01 p < 0.01 p < 0.01	Moderate correlation Very Strong correlation Moderate correlation All correlations are highly significant.

Further analysis investigated the relationship between performance on mathematical word problems (in English) and language proficiency in Gaeilge for Gaeilgeoirí (see Table 5.5). This was particularly significant at the primary to second level transition where a strong relationship was evident for all Gaeilgeoirí (r = 0.65). However, this was not replicated at the transition to third level where moderate relationships were found not to be significant for either of the groups.

Table 5.5 Correlations between mathematics performance (in English) and Gaeilge language proficiency.

	Groups & Pearson's Correlation	*Significance*	*Description*
Primary – Second Level	BG-T1: r = 0.651 BGt-T1: r = 0.706 BGc-T1: r = 0.605	p < 0.01 p < 0.01 p < 0.01	Moderate correlation Strong correlation Moderate correlation All correlations are highly significant.
Second –Third Level	BG-T2: r = 0.226 BGt-T2: r = 0.470 BGc-T2: r = 0.462	p > 0.05 p > 0.05 p > 0.05	Weak to moderate correlations but they are not significant.

[3] T = Total Group: inclusive of bilingual and monolingual students. M = Monolingual Students. BG = Bilingual Group.

For primary level Gaeilgeoirí in the transition to English-medium second level mathematics education Gaeilge language proficiency (the language of learning) was found to be of more significance than proficiency in English. Also at this transition, Gaeilgeoirí's performance on the English version of the mathematics test was highly correlated with their performance on the Gaeilge version of the test. This is consistent with Cummins' Developmental Interdependence Hypothesis (1979) which proposes the greater the level of academic language proficiency in a student's first language will allow for a stronger transfer of skills across to the new language of instruction. This suggests that Gaeilgeoirí with a high level of proficiency in Gaeilge performed well due to a strong transfer of mathematical skills across to English.

For Gaeilgeoirí in the transition to English-medium third level education a more significant relationship was found between English language proficiency and performance on mathematical word problems. No significant relationship between Gaeilge language proficiency and mathematics performance on word problems was found at this transition. Gaeilgeoirí with low proficiency both in English and in Gaeilge on average performed the poorest on the mathematical word problems. The findings suggest that developing mathematical literacy through the medium of Gaeilge at primary level will enhance the transfer to English-medium mathematics education at second level. However, this is not followed through at Gaeilge-medium second level mathematics education. The findings at this transition provide support for developing mathematical literacy through the medium of English at second level education in order to facilitate the transition to English-medium third level mathematics education. The differences in the two sets of data imply that a change in language emphasis occurs over time and that learning through the medium of Gaeilge at primary level and through the medium of English at second level may enhance mathematical learning for Gaeilgeoirí. However, the most significant overall finding at both transitions is the support for Cummins' (1976) Threshold Hypothesis. In both transitions language proficiency groups were identified and those with a high proficiency in both languages outperformed their monolingual peers, those dominant in one language and those with low proficiency in both languages (see Fig. 5.2 and Fig. 5.3). The dominant group also includes the monolingual English students. Cummins' Threshold Hypothesis (1976) does not distinguish between languages, but argues for the effect, either positive or negative in cognitive outcomes, of the

interplay of languages. Also, bilingual students displaying low proficiency in both languages were mathematically weak and lagged behind their peers. These results are consistent with the findings of Dawe (1983) and Clarkson (1992) that also draw on the work of Cummins.

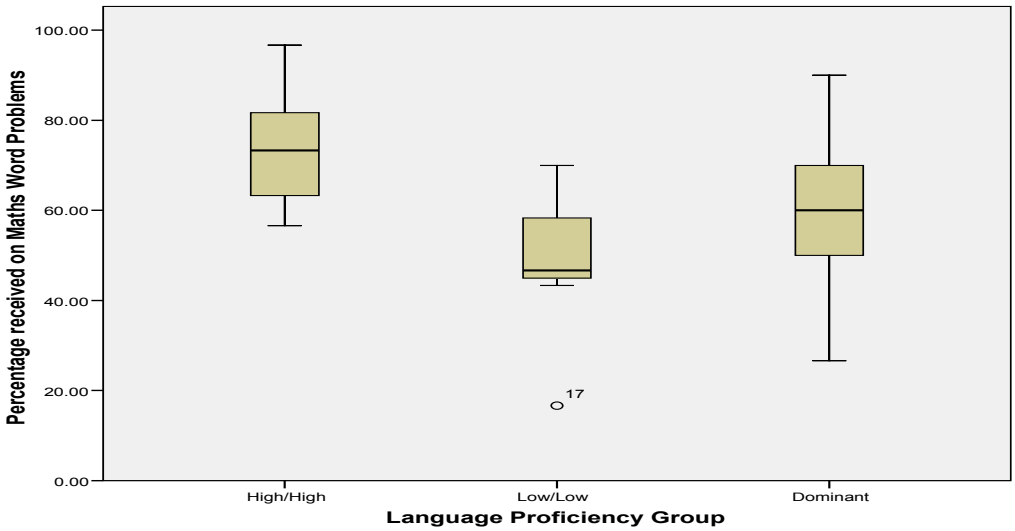


Figure 5.2 Comparison of language proficiency groups with mathematics performance (in English) at second level education.

For Gaeilgeoirí in transition to English-medium second level education (Fig. 5.2) Mann-Whitney U tests in each case showed that the difference in mathematics performance on the word problem tests is significant between the High/High proficiency group and Low/Low proficiency group, between the High/High and Dominant proficiency groups and between the Dominant and Low/Low proficiency groups

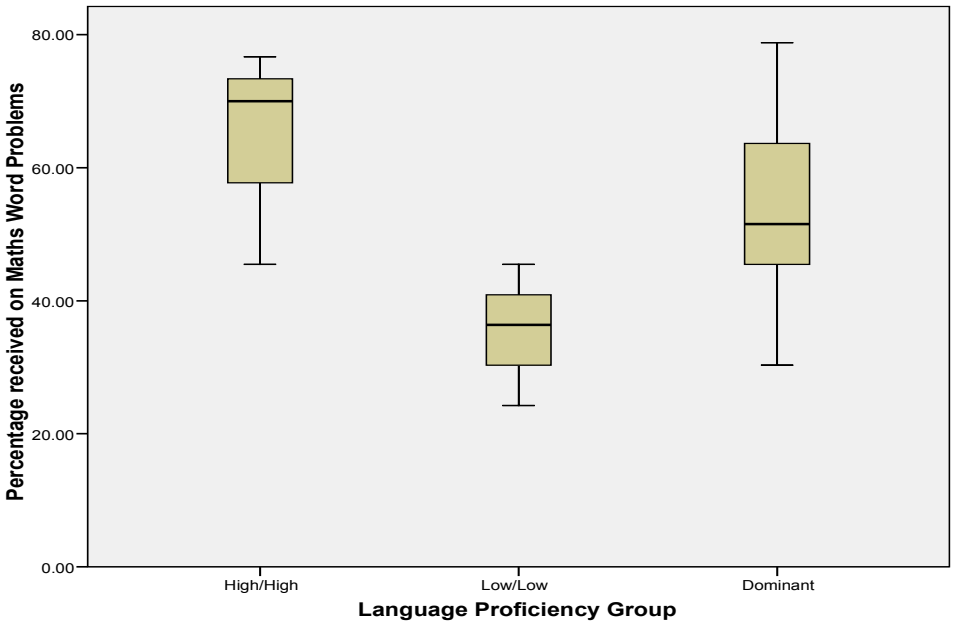


Figure 5.3 Comparison of language proficiency groups with mathematics performance (in English) at third level education

At the transition to third level education significant differences in performance on mathematical word problems were found between the High/High and Low/Low proficiency groups, and between the Dominant and Low/Low groups (Mann-Whitney U). All other differences were not statistically significant.

However it is worth noting that the bilingual students who were dominant in English performed slightly better than their monolingual peers and better than bilingual students dominant in Gaeilge. This suggests that these students had not reached the threshold level necessary in Gaeilge in order to reap the cognitive benefits from being bilingual evident for those with high proficiency in both languages The monolingual group in turn performed better than the bilingual students dominant in Gaeilge. Therefore, the greater level of English language proficiency may facilitate a stronger transfer of mathematical skills to the new language of learning (English) at third level for Gaeilgeoirí.

The findings demonstrate that language proficiency and mathematics education are related. The work carried out reveals that bilingualism is not a problem and may

enhance mathematics performance on word problems if Gaeilgeoirí have developed an adequate proficiency in both languages. The characteristics specific to the Irish education context include first language proficiency (Gaeilge) and this is a key aspect in success in mathematics learned in a second language (English) at second level education for Gaeilgeoirí. This reinforces similar findings from other countries such as Scotland (Johnstone et al, 1999) and Wales (Williams, 2002) where Maintenance Heritage Language and Immersion Education are established. At second level Gaeilge-medium education a language shift occurs and for Gaeilgeoirí transferring to English-medium third level education English language proficiency is of more importance for a successful transition. This is consistent with findings from similar educational contexts such as New Zealand (Neville-Barton & Barton, 2005) where second language learners experienced a disadvantage of between 10-15 percent in mathematics learning due to English language difficulties (their second language). A characteristic feature of both transitions in the Irish context was that low proficiency in both languages could be a significant factor in hindering learning in mathematics for Gaeilgeoirí, but given that this research was undertaken at transition points further investigation is needed to assess if these students adapt to their new learning context and catch up at a later stage. Clearly, Gaeilgeoirí face the challenge of developing an adequate proficiency both in the English and Gaeilge languages, as high proficiency in both may enhance mathematical performance on word problems as suggested by the findings.

The key findings from Research Question 1 provide support for Cummins' Threshold Hypothesis (1976). The author's work replicates that of Dawe (1983) and Clarkson (1992) whose findings provide evidence and support for Cummins' Threshold Hypothesis (1976) within a mathematics education context. Thus the author's work is contributing to the robustness of international findings, while validating the importance of Cummins' work in relation to bilingualism and mathematics education. This hypothesis has been investigated extensively at primary level education, a little at second level education but rarely at third level education. Thus the work undertaken by the author contributes to the development and support of this hypothesis beyond primary level given that the research was undertaken at second and third level education in Ireland. Very little research has been undertaken in the area of mathematics learning and bilingualism at third level education (Neville-Barton &

Barton, 2005). Therefore, this research provides a contribution to this area of research in mathematics education, as well as providing a foundation for future research to be carried out.

5.6 Key findings in relation to Research Question 2

Research question to be addressed:

2. ***Is there a significant difference in mathematical learning through English between Gaeilgeoirí transferring from Gaeltacht schools (subtractive bilingualism) and Gaeilgeoirí transferring from Gaeilge-medium schools (additive bilingualism)?***

The type of primary school attended impacted on the transition to English-medium education. Those who had attended a Gaelscoil (Immersion Education) on average performed better mathematically than those who attended a Gaeltacht primary level school (Maintenance Heritage Language). Students transferring from Gaelscoileanna dominate the High/High proficiency group, whereas Gaeltacht students dominate the Low/Low proficiency group. Given that the high proficiency group mathematically outperformed the low proficiency group significantly, it suggests that additive bilingualism (immersion education – Gaelscoileanna) enhances bilingual students' cognitive ability at this transition stage in education. Also when comparing Gaeltacht students' performance in mathematics to Gaelscoileanna students' it is evident that the latter are mathematically superior to their peers (Figure 5.4). This difference in performance is statistically significant (Mann-Whitney U, $p < 0.05$). The difference between Gaelscoil students and monolingual students' performance in mathematical word problems was also found to be significantly different (Mann-Whitney U, $p < 0.05$). However this was not the case for the performance between Gaeltacht students and monolingual students, where no statistically significant difference was found, although the Gaeltacht students performed slightly better than their monolingual peers.

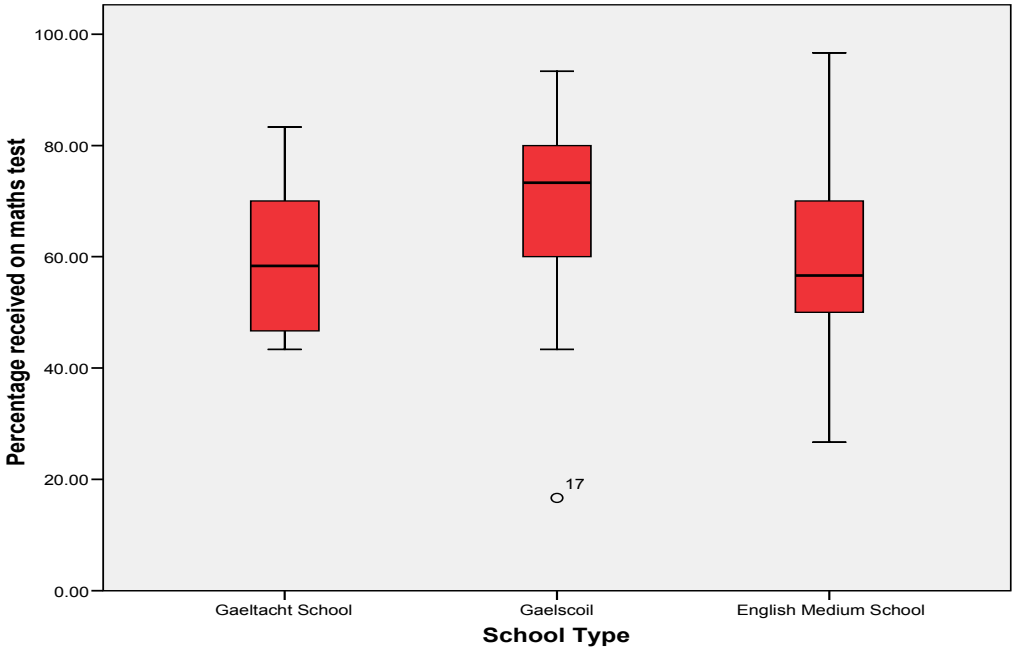


Figure 5.4 Type of school attended and mathematical performance on word problems at second level education.

Similarly at the transition from Gaeilge-medium second level to English-medium third level education, Gaeilgeoirí emerging from Gaelcholáistí (Immersion Education) performed slightly better than Gaeilgeoirí transferring from Gaeltacht second level schools (Maintenance Heritage Language schools), although it was not found to be significant. Gaelcholáiste students performed somewhat better mathematically than their Gaeltacht peers (Fig. 5.5). Although there are outliers present in the data obtained from the Gaeltacht students, from looking at these cases, case 10 and case 15 are both in the high language proficiency group, which correlates with their high mathematics performance, and case 14 is in the Low proficiency group which corresponds with their low performance on the mathematics word problem test. In turn, the low mean for this group may be attributed to the significant number of Gaeltacht students dominant in the Gaeilge language, which may have affected their mathematics performance in English. So, once again additive bilingualism (immersion education – Gaelcholáistí) is preferable to subtractive bilingualism (Gaeltacht students) as students emerging from additive learning bilingual environments performed slightly better on the mathematics word problem test than

students from Gaeltachtaí entering subtractive learning environments at third level. However, the difference in the performance between the two groups is not significant, thus further investigation is needed into additive and subtractive bilingualism at upper secondary school/third level education in Ireland. It is also evident that the monolingual students performed mathematically better than both the Gaeltacht students and better than the Gaelcholáiste students and this suggests that on a whole Gaeilgeoirí may be experiencing a slight disadvantage in mathematics on entering third level education. This is consistent with international findings which found that second language learners under perform in mathematics due to language proficiency in the new language of learning (e.g. Barton, 2003; Neville-Barton & Barton, 2005). However the difference in performance is not statistically significant between the school types.

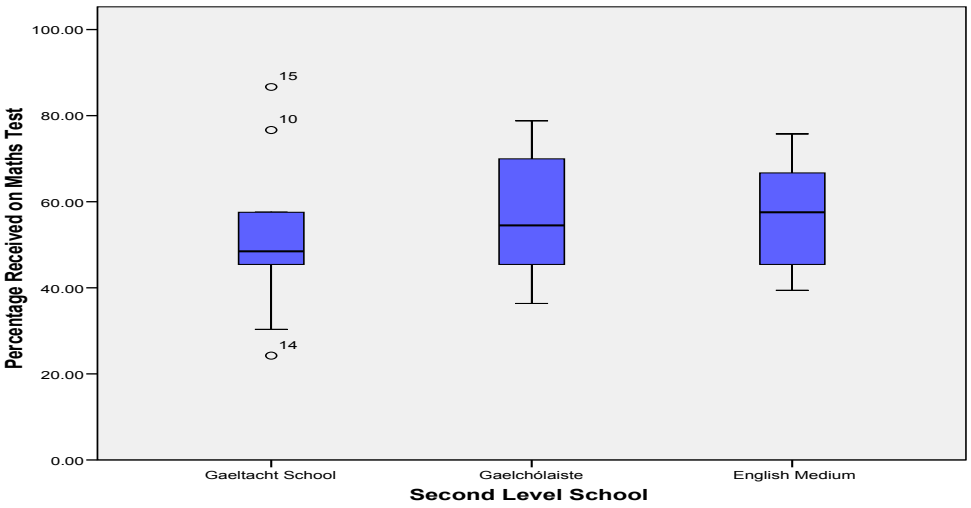


Figure 5.5 Type of school attended and mathematics performance on word problems at third level education.

The findings at both transitions in Irish education provide support for the development of *Additive* bilingualism as promoted by Immersion Education, whereas *Subtractive* bilingualism may actually be hindering Gaeilgeoirí's transition to English-medium mathematics education. Gaeilgeoirí emerging from Gaelscoileanna tend to have been immersed in both languages and encounter both languages on a daily basis. Thus the majority of these students will have developed additive bilingualism. This type of bilingualism has been shown to enhance mathematical learning (Bournot-Trite &

Tellowitz, 2002; Swain, 1996; Turnbull *et al.*, 2000). Whereas for students attending Gaeltacht primary schools, Gaeilge is the dominant language of the community and the home and therefore they do not use English, particularly academic English, for a considerable part of the time. Thus these students may experience subtractive bilingualism when they enter second level English-medium education. The literature, and the findings from this study, demonstrates that subtractive bilingualism may impact negatively on mathematics learning (Adetula, 1990; Hans & Ginsburg, 2001; Marsh *et al.*, 2000).

It is important to note here that attention needs to be paid to possible school and teacher effects. Students attending immersion education tend to be from middle to upper class backgrounds. Similarly, students attending Gaeltacht schools tend to be from a range of backgrounds with lower to middle class the dominant social class. This naturally may have an influence on the findings emerging from this study. Also, given the students attending the Immersion schools are primarily from an English speaking background, the teachers in the schools may use some English to explain certain concepts and help understanding, although the mission statement of Immersion education in Ireland is that all instruction takes place through the medium of Gaeilge. There is no research carried out to date assessing the language skills of the teachers employed in Gaeilge-medium schools. There is a possibility that they may have weak Gaeilge language proficiency and thus may employ English-medium instruction when necessary, which may have a bearing on the results emerging. The author did not look at these aspects when undertaking her research but would like to acknowledge their potential influence on the results emerging from the analysis of the data. Future work will need to address these issues in order to fully understand the situation that exists in Ireland.

However, the findings emerging from the two different Gaeilge-medium education contexts have significant implications for Gaeilge-medium education in Ireland. It is the first investigation carried out in Ireland in relation to additive and subtractive bilingualism, even though Gaeilge-medium education has been officially established since the late nineteenth century. Thus the research undertaken provides the first insight into the situation that exists in Ireland. The findings demonstrate that bilingualism does not hinder mathematical learning for some Gaeilgeoirí (Immersion

Education) but there is a need to cater for Gaeilgeoirí (Gaeltacht students) who may experience subtractive bilingualism as it may affect their mathematical learning initially when transferring to learning mathematics through the medium of English. Overall the findings provide support for Gaeilge-medium mathematics education once additive bilingualism has been achieved through the appropriate development of academic language proficiency both in Gaeilge and in English (Cummins, 1976). This is consistent with the findings from Research Question 1 in which Gaeilgeoirí with high proficiency in both languages performed better mathematically than Gaeilgeoirí with low proficiency in one or both languages and their monolingual peers.

5.7 Key findings in relation to Research Question 3

The research question to be addressed in the following sections is:

3**.** *Do particular features of the English mathematics register cause difficulty for Gaeilgeoirí at first year second level education and first year undergraduate education?*

When assessed through the medium of English Gaeilgeoirí (primary-second level interface) in this study experienced a disadvantage of 8.7% in performance on mathematical word problems. This is consistent with the findings from Phase 1 of this research project where Gaeilgeoirí performed better on the more difficult mathematics word problems when presented through the medium of Gaeilge. Improving language proficiency in English may improve Gaeilgeoirí's performance in mathematics through the medium of English. Students in the transition from Gaeilge-medium primary level education to English-medium second level education were administered the mathematics word problem test both in English and in Gaeilge.

Table 5.6 displays Gaeilgeoirí's performance in English and in Gaeilge on each of the questions and subsections (thirty answers in total). The table indicates that there were four questions in which there was no difference in performance (Questions 1(iii), 1(iv), 4 and 9 (iv)) and one question (Question 8) in which Gaeilgeoirí performed better in English. However, there were nine questions in which Gaeilgeoirí performed 10% or higher in Gaeilge as compared to their performance in English (Questions 3 (ii), 3 (iii), 5, 7 (i), 9 (iii), 10 (v), 12 (i), 12 (ii) and 12 (iii)). Overall, there was a difference of 8.7% in performance between the English and Gaeilge mathematics

word problem test, with Gaeilgeoirí performing better in the Gaeilge version. This difference is slightly lower than the findings of Neville-Barton & Barton (2005) in New Zealand, where they found that English as a Second Language (ESL) students experience a disadvantage of between 10-15% due to language difficulties. However, the finding has significant implications as it suggests that Gaeilgeoirí may not be achieving their maximum potential in mathematics when assessment is through the medium of English. Also, given that Gaeilgeoirí at this transition stage on average performed better than their monolingual peers through the medium of English, and yet are not achieving their potential level of mathematics performance when assessed through the medium of English, the difference in performance between bilingual and monolingual students may be more significant if language is taken into account. The challenge lies predominantly with mathematics teachers when assessing these students. Assessment may need to take place in a student's first language initially until adaptation to the new language of instruction and learning has taken place to ensure that assessment is valid and reflects the student's mathematical ability.

Table 5.6 Percentage of correct responses to each mathematics question in English and in Gaeilge at the transition to second level education.

Question No.	% CORRECT RESPONSES		DIFFERENCES (%)
	ENGLISH	GAEILGE	
1 (i)	89.2	94.6	5.4
1 (ii)	86.5	89.2	2.7
1 (iii)	91.9	91.9	0.0
1 (iv)	89.2	89.2	0.0
2 (i)	97.3	100.0	2.7
2 (ii)	94.6	100.0	5.4
2 (iii)	97.3	100.0	2.7
2 (iv)	94.6	100.0	5.4
3 (i)	73.0	78.4	5.4
3 (ii)	24.3	70.3	46.0
3 (iii)	40.5	81.1	40.6
4	70.3	70.3	0.0
5	21.6	40.5	18.9
6	37.8	43.2	5.4
7 (i)	27.0	37.8	10.8
7 (ii)	35.1	40.5	5.4
8	83.8	81.0	-3.8
9 (i)	83.8	91.9	8.1
9 (ii)	83.8	91.9	8.1
9 (iii)	62.2	73.0	10.8
9 (iv)	81.1	81.1	0.0
10 (i)	75.7	81.1	5.4

10 (ii)	78.4	83.8	5.4
10 (iii)	54.1	62.2	8.1
10 (iv)	73.0	81.1	8.1
10 (v)	70.3	83.8	13.5
11	75.7	81.1	5.4
12 (i)	16.2	27.0	10.8
12 (ii)	16.2	29.7	13.5
12 (iii)	29.7	40.5	10.8

(Negative value in the Difference column indicates a better performance in English)

Some of the language features of the mathematics register in English that were sources of difficulty for Gaeilgeoirí at English-medium second level education included syntax, semantics and mathematics vocabulary. Gaeilgeoirí at this transition also experienced difficulty converting digits into written vocabulary. For example in Question 3, there was a significant difference in performance between the English and Gaeilge version of the two parts of the question (24.3% vs. 70.3% and 40.5 vs. 81.1%). The syntax of the Gaeilge version lends itself to a clearer understanding of what a "Highest Common Factor" is. In Gaeilge it reads "It is called the Highest Common Factor the number that is highest, which is 4" compared to "The highest of these, called the Highest Common Factor, is 4" . The difficulty experienced by Gaeilgeoirí in answering this question correctly in English is likely due to a misunderstanding of the definition of a Highest Common Factor. Question 5 was also answered significantly better by Gaeilgeoirí through the medium of Gaeilge (40.5% answered it correctly) in comparison to the medium of English (21.6% answered it correctly). Mathematics vocabulary in English appears to be the primary source of difficulty in this question. Gaeilgeoirí may have been confused by the words "multiple" and "multiply" and may have been unsure of the difference in meaning. Whereas in Gaeilge two dissimilar words are used – "iolraí" (multiple) and "meadú" (multiply), thus lessening the confusion when interpreting and answering the question. An interesting finding from the study was that Gaeilgeoirí at this transition performed better than the monolingual students on questions involving set notation and abstract concepts associated with elements within these sets. This suggests that Gaeilgeoirí may have a more developed mode of abstract thinking than their monolingual peers. Further investigation into the mathematics register in Gaeilge may reveal a deeper insight into this proposition.

The language features of the English mathematics register that are sources of difficulty for Gaeilgeoirí at third level included syntax, semantics and mathematics vocabulary also. From Figure 5.6 it can be seen that bilingual students performed significantly poorly on fifteen of the thirty three questions - Question 4, Question 6, Question 9, Question 11, Question 12, Question 13, Question 15, Question 17, Question 18 and Question 19 (iii) - with less than 50% of the students providing correct answers to these questions. Similarly, the monolingual students performed significantly poorly on twelve of the thirty three questions - Question 9, Question 13, Question 14, Question 15, Question 16, Question 17, Question 18 and Question 19. Again less than 50% of the group got these questions correct. Therefore, Questions 4, 6, 11 and 12 appear to have been sources of difficulty for Gaeilgeoirí in this transition in comparison to the monolingual group's performance on them. Question 4 and 12 are concerned with probability and some of the Gaeilgeoirí may not have been familiar with the mathematics vocabulary employed in this question and thus affected their performance on the question. It may be worthwhile investigating further Gaeilgeoirí's understanding of probability as these were the only two probability questions on the test instrument and they performed poorly in both.

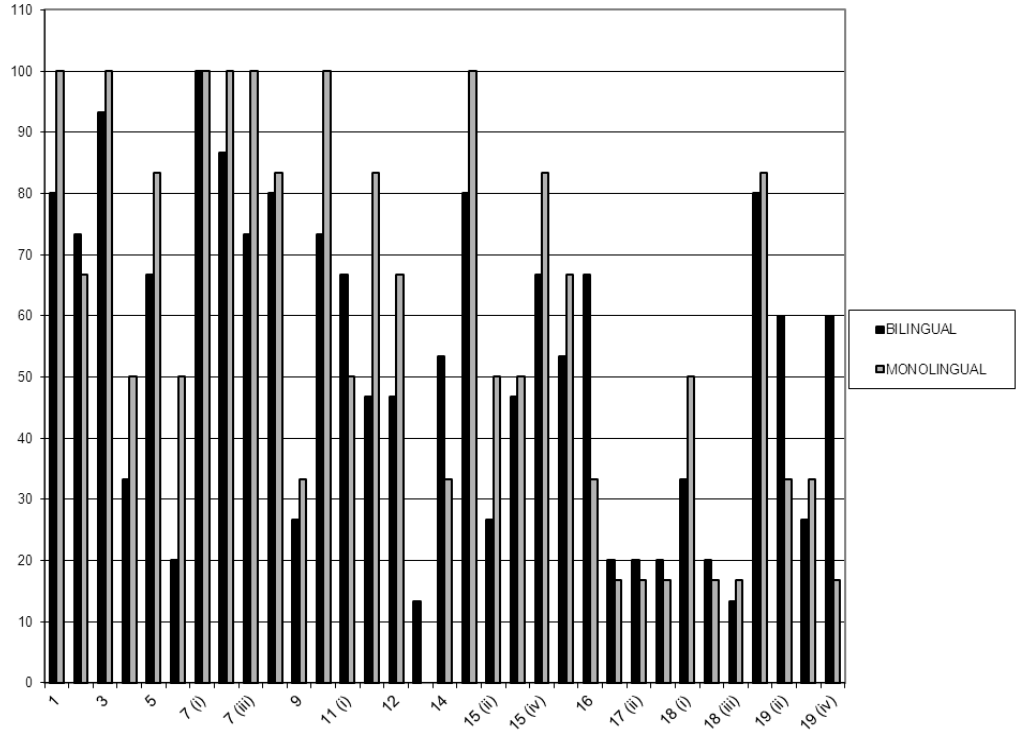


Figure 5.6 Comparison of correct responses by bilingual and monolingual students at the transition to third level.

The vocabulary and syntax of Question 6 may have been a source of difficulty for Gaeilgeoirí. In particular, Gaeilgeoirí may not be familiar with the mathematical words "numerator" and "denominator". Although these are fundamental words of mathematics, Phase 1 of this research project found that it was the basic terminology and operations that were sources of difficulty for Gaeilgeoirí in the transition to English-medium education. Question 11 is concerned with graphing, and the semantics of this word problem and the interpretation of the data presented may have been the source of difficulty experienced by Gaeilgeoirí. Overall, the common characteristics of difficulty of these four questions were syntax, semantics and mathematics vocabulary. What was interesting at this transition was that both the bilingual and monolingual students performed poorly on the cloze type questions incorporated in the test instrument. This suggests that the student's in this study at third level education have a poor understanding of mathematics vocabulary and the mathematics register in English.

The significance of the findings outlined above lies in the potential role they can play in developing teaching resources and assessments to cater for Gaeilgeoirí in the transition to English-medium mathematics education. The findings provide us with the first insight into the potential difficulties Gaeilgeoirí may experience with the English mathematics register, and mathematics educators need to be made aware of these difficulties and accordingly cater for them in their mathematics pedagogy. The knowledge of the difficulties that Gaeilgeoirí may experience with the English mathematics register in the hands of a discerning teacher can prove fruitful for easing the transition to English-medium mathematics education.

The findings emerging from the Irish context are consistent with those found in other bilingual contexts such as in New Zealand (Neville-Barton & Barton, 2005) and in Malawi (Kazima, 2007). These studies found that students learning through the medium of English (their second language of learning) experienced problems with syntax, semantics and mathematics vocabulary in the English mathematics register (Neville-Barton & Barton, 2005) and that mathematics vocabulary in relation to probability is only a problem through the medium of English for Malawi students (Kazima, 2007). Neville-Barton and Barton (2005) also found that second language

learners experience a disadvantage of 10-15 percent due to language difficulties when assessed through the medium of English (their second language). Thus the author's findings validate those found in other bilingual contexts in relation to difficulties encountered with the English mathematics register. This contributes to the robustness of international findings and provides a starting point for assessing bilinguals on entering English-medium mathematics education, as well as providing a basis for developing teaching and learning resources and support measures for learning mathematics through the medium of English.

Given that language plays a significant role in the learning and understanding of mathematics, the challenge faced by Gaeilgeoirí is acquiring the mathematics register through the medium of English. However the emphasis should be placed on the iceberg analogy or the Common Underlying Proficiency (Baker, 2001: see Chapter 3). Outwardly both languages are different in conversation but internally both languages are merged and do not function independently of one another. Thus both languages contribute to, access and use a central processing unit for mathematics learning and understanding. Gaeilgeoirí are faced with the challenge of recognising and developing the *awareness* that both languages (Gaeilge and English) are of importance to their mathematics learning and can be used to their advantage for developing mathematical understanding. Language and communication are essential elements of teaching and learning mathematics. Gaeilgeoirí will be confronted with the language of mathematics when reading textbooks and worksheets, while also having to interpret and understand the mathematical language used by the teacher. Therefore, the challenge faced by Gaeilgeoirí is not in the relearning of mathematical concepts through the medium of English. Rather the challenge lies in transferring the mathematical skills and knowledge acquired through Gaeilge to the new language of instruction. Clearly mathematics teachers play a key role in this transfer of skills.

5.8 Key findings in relation to Research Question 4

Research Question 4 was primarily concerned with the second to third level transition in Irish education. Gaeilgeoirí were administered a questionnaire and they undertook an extensive interview. The primary aim was to gain an insight into how language has impacted on the transition from Gaeilge-medium to English-medium mathematics

education from a pedagogical, cultural and personal perspective. The research question to be addressed is:

4. Do cultural and pedagogical factors influence Gaeilgeoirí's transfer from Gaeilge-medium to English-medium education?

5.8.1 Methodology and Analysis of the Questionnaire

Only the bilingual students at this transition completed the questionnaires. This included students transferring from Gaeltacht secondary schools (9 in total) and from Gaelcholáistí (6 in total). Analysis of the questionnaire involved both quantitative and qualitative aspects, which is in line with the mixed methods approach employed in this study. The quantitative aspects were coded and imported into SPSS for analysis where descriptive statistics were utilized in order to assess the frequencies of the appropriate comments/responses to specific questions. The qualitative aspects were analysed using NVivo as the organisational tool.

5.8.2 Findings from the Questionnaire

The qualitative and quantitative analysis of the questionnaire was undertaken simultaneously so as to get a comprehensive understanding of the information provided by Gaeilgeoirí and thus the findings are organised under a number of key headings in order to provide a detailed description of their experience of the transition to English-medium third level mathematics education. A number of *significant insights* were generated from the qualitative analysis undertaken on the data collected at the transition from second to third level education.

The general consensus amongst the Gaeilgeoirí was that mathematics involves problem solving, the study of numbers and the different functions of numbers, and having to work out the correct solution to a given mathematical problem. Given this it was not surprising that very few Gaeilgeoirí believed that there was a relationship between mathematics and language. Clearly language does not seem of importance for the learning and comprehending of mathematics for these Gaeilgeoirí. The subjects are largely unaware of difficulties related to language, and this is consistent with international findings (e.g. Neville-Barton & Barton, 2005).

The participants were asked to rate the difficulty of a series of questions related to mathematics and language (Neville-Barton & Barton, 2005). The rating scale had four

positions: 1 = not difficult; 2 = a little difficult; 3 = difficult and 4 = very difficult. The average rating for each is given in brackets after the particular question.

Understanding the English used by other students	(1.07)
Reading the blackboard/whiteboard/overheads	(1.47)
Understanding the English used by the lecturer/tutor	(1.60)
Reading mathematics textbooks	(2.13)
Reading photocopies/handouts given by the lecturer/tutor	(1.67)
Reading tutorial/exam questions	(1.93)
Understanding the mathematics content of lectures/tutorials/exams	(1.93)

Clearly all the average ratings were in the "not difficult" to "a little difficult" categories. From the above it appears that Gaeilgeoirí are relatively confident in coping with English as their new medium for learning mathematics. However, this confidence is not reflected in their performance on the mathematics word problem test. The average performance of bilingual students at the transition to third level education was 53.73% with a standard deviation of 18.03. This suggests that Gaeilgeoirí are not aware of the influence of language on mathematics learning and that this lack of awareness is hindering them when transferring to English-medium education. This is reflected in the slight disadvantage they experience in mathematics performance in comparison to their monolingual peers (median = 57.07% with a standard deviation of 13.87).

There exists among Gaeilgeoirí a fear of acknowledging and voicing that they may be experiencing difficulties and consequently third level institutions are unaware of some of the issues Gaeilgeoirí are encountering. Thus no specific support structures are in place in any of the institutions who participated in this study. Gaeilgeoirí are relying on procedural knowledge in order to pass examinations, with little time given to the development of conceptual understanding. Some of these students are suffering loss and displacement due to having to study mathematics through the medium of a new language. This loss and displacement is occurring in mathematics understanding. The consequence of this may be their exclusion from the skills base of the emerging knowledge society.

Throughout the questionnaire students rarely made reference to the fact that they have two languages (Gaeilge and English) and thus are bilingual. When questioned on their opinion of having studied mathematics through two languages only two of the students could see an advantage in being bilingual. However the remainder of the students felt that bilingualism had no positive effect on their mathematics learning, in particular citing the lack of use of Gaeilge and the dominance of English at third level. Overall they had a very negative view of bilingualism in that the transition to English-medium mathematics education caused confusion and difficulty, and that "You have to learn off the terminology again through English" (Eiméar). There was little recognition on their behalf of transferring skills from Gaeilge to English, constant reference was made to relearning through English with some of the mathematics acquired through Gaeilge becoming redundant once they had finished their second level education.

5.8.3 Methodology and Analysis of the Interviews

Follow-up interviews were undertaken with seven Gaeilgeoirí who had completed the mathematics word problem test, English and Gaeilge language competency tests and the questionnaire. The interviews were conducted in order to obtain an understanding of the language employed by Gaeilgeoirí when immersed in mathematical problem solving, to establish their perceptions of mathematics and of mathematical learning, while appraising their experiences of the transition. This section of data collection was primarily qualitative in nature. Semi-structured interviews were conducted with the seven Gaeilgeoirí, which were digitally recorded and transcribed. During the interview Gaeilgeoirí completed a Language Use Survey (Clarkson, 2007) and two mathematics word problems (Newman Research Method, 1977), along with responding to questions based on specific themes. The analysis was largely qualitative and NVivo was utilized for this purpose. Quantitative analysis (SPSS) was employed for evaluating the Language Use Survey.

The author decided that the analysis of the Newman Method would remain a separate unit to the analysis of the remainder of the data, but naturally a relationship between findings would emerge and are discussed subsequently. Six prominent themes emerged from the data. These were Culture, Pedagogy, Mathematics Understanding, Bilingual Factors, Conceptions of Mathematics and Language Use. Four of the above

are consistent with the themes that emerged from Phase 1 of this research project (see Fig. 5.1). Thus the model generated from the Maths Life Histories in Phase 1 has been altered and adjusted, consistent with the findings emerging from the interviews undertaken in Phase 2 of this research project. This new model (Fig. 5.7) is presented on the next page and the relationships between the relevant nodes will be discussed.

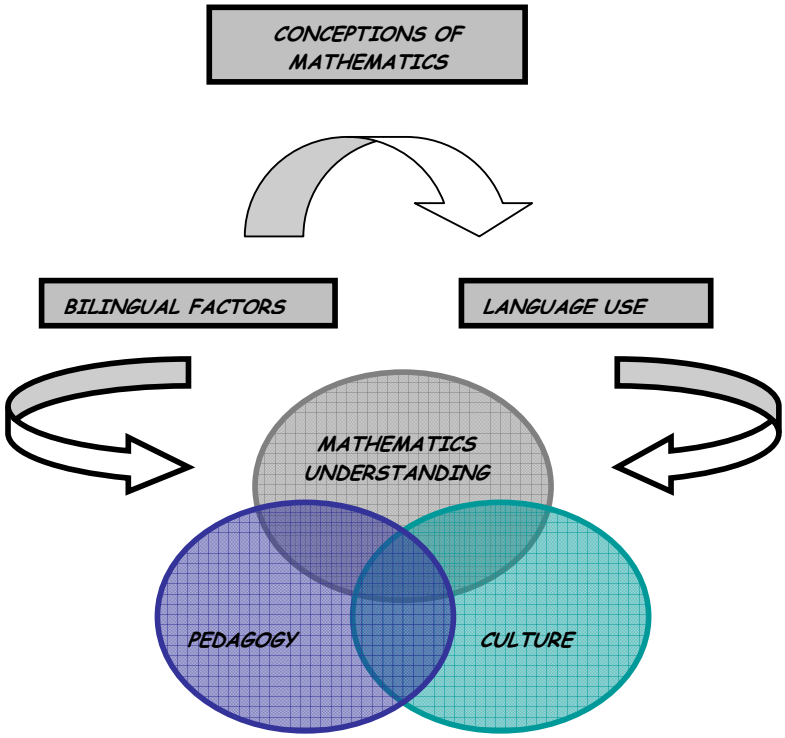


Figure 5.7 Diagrammatical representation of the interview findings.

Findings from the Interviews

The first relationships to be discussed are that of the triad of Mathematics Understanding, Pedagogy and Culture. They are examined in pairs initially with a final discussion on the group as a whole.

Mathematics Understanding – Pedagogy

"Understanding the question and being able to interpret what is being asked. So yeah being able to answer the question fully and correctly." (Síle).

The general consensus when questioned on what they believe "understanding mathematics" to be was one of an instrumental perspective (Skemp, 1978). Being able to solve mathematical problems using mathematical formulae and correct procedures reflected one's ability at understanding mathematics. Granted instrumental understanding plays a significant role in mathematics learning but none of the participants interviewed relayed an importance of knowing "why" (Skemp, 1978). Clearly relational understanding is not of significance in their mathematics learning.

However, pedagogical aspects need to be taken into consideration when engaged in a discussion on learners' perspectives of mathematics. This became prominent in discussions on teaching strategies employed by the participants' mathematics lecturers/tutors. The primary teaching method employed by mathematics lecturers was one of dissemination of knowledge through the use of examples. Little time was devoted to theoretical aspects or engagement in problem solving during the course of a lecture. The constraint of taking down a large amount of examples, as well as the speed of the delivery of the lecture gave little opportunity for listening or cognitive involvement in the lecture. As Sinéad summed up:

"You can't really do the two (note taking /listening). So if you're taking stuff down you don't really understand what he's doing because you can't listen at the same time 'cos there's so much to take down."

Similarly during tutorials emphasis was placed on the practice of a sheet of questions given by the lecturer. Tutorial sheets were structured in such a way that the students gained experience of practicing similar questions, as well as practicing the *methods* required for solving different types of questions.

Clearly, mathematics lecturers/tutors are placing an emphasis on procedural knowledge for the learning of mathematics and little time is given to the development of conceptual understanding. Because of this emphasis Gaeilgeoirís' learning strategies revolve around the continual practice of tutorial questions and previous exam papers. Rote learning is relied on and fostered through success in examinations. Thus the cycle continues and students proceed unaware of the gaps in their

mathematical knowledge and therefore it is not surprising that Gaeilgeoirís' perceptions of mathematics understanding reflect a procedural nature. The system of mathematics teaching in Ireland at second and third level education cultivates this type of perspective, and success is achieved through rote learning and relaying of learned methods. So for Gaeilgeoirí, their acuity for procedural methods in learning and "understanding" mathematics is rewarded by a system that instinctively promotes this type of knowledge. The pedagogical system fails to encourage relational understanding and thus appreciation of the importance of this type of knowledge is lacking in their perceptions of mathematical understanding.

Mathematics Understanding – Culture

Learning (and consequently understanding) is an incessant and enduring process that is a consequence of partaking in a formal or informal learning environment. Gaeilgeoirí entering Irish third level education are emerging from a learning environment immersed in the Irish language and culture. They are required to adapt to a learning environment steeped in the English language and cultural practices. Therefore Gaeilgeoirí are required to use mathematical tools within this new environment with little regard given to previous language/cultural practices. The assumption is that all students have done mathematics through English. For the Gaeilgeoirí who participated in this study, naivety was the dominant trait on entering English-medium mathematics education.

It was something that never occurred to them when at second level education, nor was it a topic broached by their teachers. Naturally the transition to English-medium learning was a "shock" and "difficult" transition for some of the Gaeilgeoirí. Learning mathematics through the medium of a new language impacted on the majority of these Gaeilgeoirí and this will be discussed in greater depth in the section on mathematics understanding and language.

However, cultural difference (which does include language) played a significant role in the transition for these students. Many felt that the other students in their courses perceived them as being different.

"Sometimes when in class I stand out a little bit because my English isn't up to their level…. They'd know like because my accent and stuff that I'm different"

(Tomás).

This in turn prevented them from engaging in discussions that arose during tutorial times. They lacked confidence in their ability to partake in mathematical discussions and ask questions through the medium of English.

The lack of use of Gaeilge for academic and social purposes at third level was evident from the interviews. Lecturing or tutoring never incorporated the language. This was a significant factor for these Gaeilgeoirí as this was the medium they had learnt mathematics through for 13-14 years of schooling. They also still relied on Gaeilge when engaged in mathematical practice (see section on language use) and thus the impact of Gaeilge on their mathematics learning cannot be ignored. Unfortunately for many Gaeilgeoirí they feel the pressure to succumb to the "norm" of using English and reduce the amount of Gaeilge they speak/use – "Yeah I definitely feel like I'm different and now I have to cut out the amount of times I use Gaeilge in college because of this" (Síle). One would question if this were having a negative impact on their mathematics understanding given that this was the primary language of learning for these students. Although complex and often obscure, cultural procedures can be acquired quickly and with ease when one is immersed in the milieu and this may explain Gaeilgeoirís' adaptation to the new language and cultural environment of learning (Brown, Collins & Duguid, 1989). They all referred to the fact that "you got used to it after a while." (Colm).

Pedagogy-Culture

Too often mathematics students are required to use procedures of the discipline without acquiring and embracing the culture of the discipline. If one is to embrace a mathematics culture, then learning is a process of enculturation (Brown, Collins & Duguid, 1989). For example, the regimented use of mathematical formulae by students is rather different from the way (mathematical) practitioners employ them.

This is reflected in the fact that students can pass exams (and therefore appear to understand mathematics) but yet they lack the conceptual understanding of procedures utilized.

However all students interviewed highlighted the didactical approach employed by many lecturers/tutors at third level education in Ireland. Gaeilgeoirí were presented with abstract concepts and independent examples. Thus Gaeilgeoirí were exposed to the procedural tools of mathematics but lacked "authentic activity" in order to truly understand the conceptual tools being employed (Brown, Collins & Duguid, 1989). It appears that many lecturers at third level education lack the pedagogical skills necessary to facilitate enculturation into the mathematics domain. What was interesting in the students' interview responses were their apparent lack of an ability to critique the teaching methods employed by their lecturers/tutors. The teaching style employed by their lecturers was perceived as being the norm and accepted method, and they found it difficult to suggest ways in which they could improve their teaching methods. The primary response was always "to give more examples" and "to go slower through the steps of the examples". Clearly this would facilitate their rote learning and practice of questions, Gaeilgeoirís' key learning process.

Gaeilgeoirí were quick to highlight the positive aspects of entering third level education such as independence, choice of study, relevance of course of study, and the social elements associated with university education. Conversely, the culture of third level institutions is to promote independent autonomous learners which the students interviewed found difficult adapt to. There was a lack of interaction between lecturers and students in comparison to the more individual attention received at second level. Some felt that lecturers were unapproachable and would "get my friend to help me if I was having difficulty" with the mathematics (Liam). There appeared to be a fear of being perceived as "different" and "weak", undesirable traits of Irish third level cultures. Also, Gaeilgeoirí believed that there would be a lack of understanding on the behalf of the lecturer/tutor as they would not be able to relate to what they were experiencing (i.e. change in the language of instruction/learning). Clearly some third level institutions are impeding the mathematics enculturation process through the pedagogical practices adhered to.

Mathematics Understanding – Pedagogy – Culture

The previous three sections have emphasized the relationship between the various pairings within the triad. Obviously there is a relationship between the three components of the triad. In order for Gaeilgeoirí to develop mathematical understanding, pedagogical practices are of key importance, which in turn are evocative of cultural influences. Mathematics is a product and a function of culture and unfolded through pedagogy. Mathematics understanding, pedagogy and culture are interdependent in the triad. Neither can be understood without the other.

The next stage of analysis will look at the upper section of the model, which consists of the quartet of Mathematics Understanding, Bilingual Factors, Conceptions of Mathematics and Language Use. Once again the relationships between each pair will be explored and discussed based on interview data gathered.

Mathematics Understanding – Bilingual Factors

"Going from Gaeilge to English is hard at times…it has made the transition to college mathematics difficult for me but I'm getting used to it." (Colm).

Although the majority of Gaeilgeoirí interviewed found the *general* transition from second to third level education relatively easy, having to transfer from learning mathematics through Gaeilge to learning mathematics through English impacted on their learning and understanding of the subject. Ignorance on their behalf of the transition they faced meant that some of those interviewed found the language shift difficult initially. The main source of difficulty was the actual 'language of mathematics' i.e. the mathematics register. This was very evident from discussions on the test instrument they completed. In particular the final three questions in which they had to complete cloze type mathematics questions were completed poorly. None of the participants achieved high marks in these questions. The following reflects Gaeilgeoirí's perspective on those questions:

"I didn't really know what words to put in there. I think it was because of learning through Gaeilge. I would have known the terms and their meaning through Gaeilge so I didn't know them through English." (Sinéad).

The cloze mathematics questions consisted of some fundamental terminology of mathematics e.g. integer, odd number, prime number but given that Gaeilgeoirí had learnt these terms through Gaeilge at primary level and perhaps had not encountered the English version of the words at third level they were unable to complete the questions. They all referred to problems they encountered with mathematics terminology they had acquired at primary and second level through the medium of Gaeilge but were unaware of the English equivalent on entering third level. They spoke of the expectation on the behalf of lecturers/tutors that all students had learnt their mathematics through English and thus their learning needs were not catered for in the transition.

The above perspective is reflected in their suggestions for improving teaching strategies at third level in order to cater for bilingual learners. The suggestions included being able to continue with learning mathematics through the medium of Gaeilge in first year (in particular for those who take mathematics at service level). Others suggested having bilingual notes and a Gaeilge version of an exam paper, and there was strong support for the proposition of lecturers writing the Gaeilge version of the word next to the English word used in their overheads/lecture notes because

"..you would recognize what they are doing and make the connection and it would help with your understanding because you would know the word in Gaeilge so you would know what they are talking about." (Síle).

Although the participants were experiencing some difficulty in the transition and evidently it was affecting their mathematics understanding, all spoke of how accustomed you become to using English in due course. Clearly, Gaeilgeoirí require assistance in the initial transition and perhaps if appropriate teaching interventions were introduced this transition process may be eased as well as improving mathematics understanding. The fact that all students interviewed are relying on rote learning in order to pass examinations and thus are seen to "succeed" at mathematics would suggest that the type of understanding being developed is not the desired one and this has significant implications down the line for future mathematics learning and career development.

Mathematics Understanding – Conceptions of Mathematics

In the previous section the participants' conceptions of mathematics were examined. It was found that the majority had a very narrow perception of mathematics in that they strongly believe it only consists of "numbers", "formulae" and "using numbers to solve problems". Few saw a relationship, relevance and importance for everyday life. The purpose of doing mathematics was solely to pass the course of study they were undertaking. This perception partly stems from the mode of teaching employed at second and third level education. Didactical teaching is the norm where repetitive practice of questions is encouraged. Thus, Gaeilgeoirí are not gaining a deeper insight into the subject area as a consequence of the teaching methods they have encountered.

What was surprising for the author (this was also evident in Phase 1 of the research project) was given the emphasis that Gaeilgeoirí placed on problems they encountered with mathematics terminology and the change in the language of learning, only two saw a relationship between mathematics and language.

"Ammm I really don't think it (language) makes a difference to mathematics to be honest because most of them aren't really wordy anyway so I don't think it makes a difference in mathematics at all." (Cliodhna).

Obviously their perception of the subject matter influences their conceptions of mathematics. If Gaeilgeoirí lack awareness of the influence of language on mathematics learning and understanding then this may have repercussions for their mathematics understanding. As outlined in Chapter 2 language plays a key role in mathematics learning and understanding and awareness of this is crucial when immersed in a new language of learning.

Two of the participants could see a relationship between mathematics and language and what was interesting about this was the fact that of the seven students interviewed these were the students who expressed the greatest difficulty with the change i.e. in the language of instruction.

"Yeah you have to understand the question that's being asked and I don't know (pause), you can't do it without language." (Síle).

Clearly they were aware that language may be a source of some of the difficulty they were experiencing. The author believes that this *awareness* of language as a source of difficulty may actually help the students' mathematical understanding in the transition from Gaeilge-medium to English-medium education. Both of these students were able to speak of how they used both Gaeilge and English when answering mathematical problems whereas the others interviewed seemed to use both languages subconsciously i.e. they lacked an awareness of the fact that they use Gaeilge when engaged in English mathematical problem solving. This will be discussed in greater depth in a subsequent section.

Mathematics Understanding – Language Use

During the interview Gaeilgeoirí were asked to complete a Language Use Survey (see Clarkson, 2007). This consisted of identifying what language(s) they used in answering the individual word problems on the test instrument. Gaeilgeoirí were given the option of selecting English Only, English and Gaeilge, or Gaeilge Only. Table 5.7 displays the findings from this Language Use Survey. From Table 5.7 it is obvious that Gaeilgeoirí drew on their first language of learning when answering some of the mathematics word problems even though all of the problems were presented in English. When questioned further on when and what language they use the majority found it difficult to explain how they employed the different languages. After further probing it became apparent that Gaeilge was used primarily for thinking out a problem and conducting mental operations such as addition and multiplication of numbers as this was what they described as "normal" and "natural" to them. Given their inability at times to describe their use of languages it appears to be a subconscious action and ingrained in their process skills when engaged in mathematical problem solving and understanding. The author raises the question as to whether bilingual instruction should be available to these students initially in order to cater for their language needs in mathematics learning and understanding given their tendency to employ both languages?

Table 5.7 Findings of the language use survey.

Question	*English Only (No. of Students)*	*English & Gaeilge (No. of Students)*	*Gaeilge Only (No. of Students)*
1	2	4	1
2	4	3	0
3	4	2	1
4	4	2	1
5	1	4	2
6	3	2	2
7(i)	4	3	0
7(ii)	4	3	0
7(iii)	6	1	0
8	6	1	0
9	1	5	1
10	4	2	1
11(i)	3	4	0
11(ii)	4	3	0
12	4	2	1
13	4	2	1
14	5	2	0
15	4	2	1
16	1	6	0
17	2	4	1
18	1	5	1
19	2	5	0

Several of the students interviewed used translation of mathematical terminology and of some phrases when answering questions. Clearly in order to employ translation Gaeilgeoirí need to know the English *and* Gaeilge version of the mathematical word(s) and therefore the author explored this concept further.

"I suppose it (translation of words) gives me confidence to do some of the mathematics…the stuff I'd have done all through Gaeilge so I'd know that stuff through Gaeilge and it'd be easier to do through Gaeilge." (Sinéad)

From the discussions it became clear that using Gaeilge when answering mathematics questions gave them a confidence to approach the question and attempt answering it. Whereas if they didn't know the Gaeilge word in order to translate it they tended to

leave the question blank rather than attempt it and get it wrong, when they were completing the test instrument. Thus the Gaeilge language and its use are still important to Gaeilgeoirí in the development of their mathematical understanding.

The language of presentation was also a significant factor in the language choice employed in mathematical problem solving. Gaeilgeoirí spoke of attempting the question through English if it was presented in that language. If it were proving difficult they would then employ Gaeilge (if possible) to help them to solve the problem. However basic mental calculations were nearly always undertaken through the medium of Gaeilge. Similarly if the question were presented in Gaeilge then they would attempt answering the problem through the medium of Gaeilge. The majority of those interviewed would prefer to continue with learning mathematics through the medium of Gaeilge predominantly because of habit and their natural tendency to undertake mathematical problem solving through this medium.

Bilingual Factors – Conceptions of Mathematics

As discussed previously Gaeilgeoirí's conceptions of mathematics revolved around the belief that it consists of numbers, problem solving and using formulae. For this reason it is understandable why the majority saw the transition from learning mathematics through Gaeilge to learning mathematics through English as 'relearning' mathematical words and concepts through the new language of instruction. This was a prominent finding from Phase 1 of this research project also. There was no recognition of transferring mathematical skills from one language to another language or drawing on skills developed in both languages to solve mathematical problems. Rote learning was employed in this relearning of previously acquired skills, as well as developing new mathematical knowledge through the medium of English. This demonstrates their lack of *awareness* of the fact that they do use Gaeilge relatively often when engaged in mathematics (see section on language use) and therefore are drawing on both languages. The general perception appears to be that if the question is presented in English then they are solving the problem through English, and likewise for Gaeilge. Because of this lack of awareness of use of language, participants saw no real advantage to having two languages for learning mathematics. In fact some saw it as "a little confusing". Perhaps if Gaeilgeoirí were more aware of

the influence of language on mathematics learning this would make the transition easier and improve their mathematics understanding.

Conceptions of Mathematics – Language Use

As examined previously Gaeilgeoirí's conceptions of mathematics fail to acknowledge a relationship between the subject and language. As a result the Gaeilgeoirí interviewed consider language competency in one or both languages as irrelevant to mathematics learning and understanding. The purpose of language is solely for reading questions but they failed to see that this is an important step in solving a mathematics problem and developing understanding.

"Just understanding what was asked of me caused difficulty." (Síle)

But Gaeilgeoirí failed to make a connection to language as a facilitator of understanding. They considered the mathematics content solely as the source of difficulty. As a consequence this may be acting as a barrier to developing their mathematical skills and understanding.

Bilingual Factors – Language Use

It was apparent from analysis of the interviews and of the Language Use Survey that Gaeilgeoirí make use of both languages (Gaeilge and English) when solving mathematical problems. However there is a clear lack of awareness of the use of both languages on the behalf of some of the students. It was only through probing the students that they began to realise that they used Gaeilge even if it was

"only just for simple things like adding and multiplying." (Liam).

Because this was what they were accustomed to using when involved in mathematics problem solving it became natural and almost a subconscious action. Again they failed to see an advantage to having two languages for learning mathematics, but perceived it as occurring in one language or the other at a given time. The author feels strongly that this lack of awareness of language use and connection with mathematics

learning may be acting as an obstacle to Gaeilgeoirí developing mathematics understanding at the transition to third level education.

Mathematics Understanding – Bilingual Factors – Conceptions of Mathematics – Language Use

The above have been discussed in pairs and the relationships between all elements have been explored. All are interdependent and none can be looked at in isolation from another. The primary aim is the development of mathematical understanding but for Gaeilgeoirí significant bilingual factors, conceptions of mathematics and language use need to be taken into consideration.

Although the upper and lower sections of the model were discussed in isolation there is a clear link between the two. What the overall model seeks to demonstrate is the complexity of the issue being investigated. There is a need for researchers and teachers of Gaeilgeoirí to be aware of the factors explored in this model as they have a significant impact on the teaching and learning requirements of Gaeilgeoirí in the transition to English-medium third level mathematics education.

5.8.4 Findings from the Newman Research Method (1977)

The last section to be looked at from the analysis of the interviews undertaken is Gaeilgeoirí's problem solving abilities through the medium of English. The Newman Research Method (1977) was employed in this section of the interview. The participants were asked a series of questions in order to establish if they could read the question, comprehend what they had read, conduct an appropriate mental transformation from the words of the problem to the choosing of a mathematical strategy, employ process skills required by the selected strategy and encode the answer in an appropriate written form. At all times Gaeilgeoirí were asked to talk about the mathematical steps they were undertaking and why, until they reached a solution.

Reading

All Gaeilgeoirí were able to read the word problems presented to them. They didn't encounter any difficulty with the phrasing and wording of the questions. However two of the interviewees were noticeably slower at reading the questions in comparison to

the other participants. They appeared to lack confidence in reading out loud through the medium of English. One needs to take into account that this may be a heightened reaction given that it was during an interview and the participants may not have felt at ease in this situation. However it may be worth investigating if the language of presentation influences Gaeilgeoirí's reading of mathematical problems, e.g. do they feel more comfortable reading questions through the medium of Gaeilge or through the medium of English.

Comprehension

Although the Gaeilgeoirí were able to read the questions given, their ability to comprehend what they had read varied between the questions. Four of the students understood what was being asked in Question 1 i.e. that you have to calculate the price of the jersey both ways in order to see if there is a difference in the price. Whereas the others assumed that the price of the jersey was cheaper being calculated one way over the other without undertaking an investigation.

"Well I want to prove that buying the jersey with 30% off, taking the 30% off first is cheaper than taking the 5% off first." (Donall).

This statement is very evocative of the misconceptions that Gaeilgeoirí encountered with the problem presented to them. Firstly the use of the word 'prove' reflects their conceptions of mathematics as consisting of numbers, proofs and working out solutions, and the ultimate goal of obtaining a correct answer. They have also misunderstood what the question is asking of them with the mix-up in the percentages to be added and deducted from the price of the jersey. The other students made similar assumptions and blunders. Given the question is relatively long and wordy, the language may have been a source of difficulty and confusion.

Also in Question 2 a number of the students had misconceptions of what was being asked. The question is concerned with probability and proved to be difficult to solve for the majority. For example one student's comprehension of probability was to employ ratios immediately in the calculation of the answer and it led to further confusion:

"You toss a coin so you've got two sides so it's either going to land on head or tails so it's fifty fifty……..How would you write that in a ratio? Is it one to one? (Tomás).

The above is also reflective of the lack of confidence some Gaeilgeoirí had in their comprehension of what was being asked. This lack of confidence may be hindering them from progressing in answering a question. There is a big fear of getting the 'answer wrong'. Gaeilgeoirí appear to lack confidence in their comprehension of mathematical problems through the medium of English.

Transformation (or Mathematising)

In order to undertake transformation Gaeilgeoirí need to comprehend the question being asked. As examined in the previous section some of the Gaeilgeoirí's understanding broke down at the comprehension stage and in turn affected the transformation from mathematical text to the choosing of an appropriate mathematical strategy for solving the problems. For example on Question 2 one student had failed to comprehend what the question was asking and this affected their selection of an appropriate method for solving the questions.

"Then you've got a dice with six sides. So six into a hundred is (pause) sixteen point something." (Tomás).

Clearly he was concerned with working out a percentage as he was thinking of the final answer he would present. Gaeilgeoirí who had comprehended any of the questions correctly also performed the correct transformation of the problem by selecting the correct strategies for solving the word problem.

Process Skills

Although Gaeilgeoirí at times failed to comprehend some of the questions asked and thus selected inappropriate mathematical strategies, their process skills reflected the (incorrect) mathematical strategies they had selected. Naturally, these process skills may not have been the ones required for solving the questions correctly but they were easily connected to the strategies they had selected themselves. The calculations required to solve the questions were not particularly difficult so perhaps this is reflective of their ability to undertake the selected process skills. The fact that their

process skills were comparatively good perhaps reflects the pedagogy employed in mathematics teaching where emphasis is placed on the *methods* for solving questions and the continuous practice of these methods. Clearly process skills are valued, with little regard given to comprehension and transformation in mathematics understanding.

Encoding

The final part of Newman's method is encoding the answer in an appropriate written form. All Gaeilgeoirí attempted providing some form of an answer. What was of interest in the probability question was their scant attention to detail as many failed to realize that it was requiring them to combine the tossing of a coin with the throwing of a die. Some calculated both individually and reported the answers accordingly. Similarly for Question 1 some calculated the cost of the jersey both ways as required but failed to report what conclusion could be drawn about the calculating of the cost of the jersey. Again it appears that their comprehension of the questions affects the final encoding of the answer.

Overall it is apparent that the key stage where Gaeilgeoirí's understanding breaks down is at the comprehension stage when engaged in solving mathematical word problems. This in turn has significant implications for the transformation of the content and the selection of an appropriate mathematical strategy for solving the problem. However where they tended to perform well was on the process skills, which reflects the system they are accustomed to and these are the skills valued in many mathematics examinations at second and third level education in Ireland. The author would also like to note that Gaeltacht students performed poorly on the questions in comparison to their Gaelcholáiste peers, as comprehension through the medium of English was not a source of difficulty for these students. This is reflective of the type of schooling they are emerging form i.e. additive bilingualism where they would have been immersed both in the Gaeilge and English languages.

5.8.5 Summary of Findings in Relation to Research Question 4

The findings of this research question demonstrate how mathematics understanding, pedagogy and culture are interdependent and neither can be understood without the other for Gaeilgeoirí. The pedagogic practices employed at second and third level

education in Ireland promote and encourage procedural knowledge with little time devoted to the development of conceptual understanding. Accordingly the system rewards rote learning and relaying of learned methods through examinations, and thus it is not surprising that relational understanding is lacking in Gaeilgeoirí's perceptions of mathematics learning and understanding. Gaeilgeoirí entering English-medium third level education are emerging from a learning environment immersed in the Irish language and culture. This had a significant bearing on their experience at third level as well as influencing their mathematics understanding, as this culture was never acknowledged nor fostered by the institutions who participated in this study. It became very apparent that mathematics enculturation was impeded through the pedagogical practices employed at third level education.

Similarly, relationships between mathematics understanding, bilingual factors, conceptions of mathematics, and language use were also established for Gaeilgeoirí. Again all are interdependent and none can be looked at in isolation. Although the majority of those interviewed found the general transition to third level education relatively easy, having to transfer from learning mathematics through Gaeilge to English posed problems for many and impacted on their learning and understanding of the subject. Their conceptions of mathematics were narrow and reflective of the procedural processes they have been subjected to. However, given the emphasis that Gaeilgeoirí placed on issues encountered with mathematics terminology and the change in the language of learning, it is surprising few saw a relationship between mathematics and language. The Language Use Survey revealed that Gaeilgeoirí employ both languages when engaged in mathematical problem solving. Gaeilge was used primarily for thinking out a problem and conducting mental operations such as addition and multiplication. This appears to be a subconscious action and ingrained in their process skills. Translation was also employed as a means to help interpret mathematical content and provide confidence in attempting to answer the question(s). However, Gaeilgeoirí seemed to lack an awareness of their language use and thus they saw no real advantage in having two languages for learning mathematics. Finally, a key finding emerging from the data gathered at the transition to third level education is that Gaeilgeoirí's understanding tends to break down at the comprehension stage when engaged in mathematical problem solving (Newman Research Method, 1977). This in turn has significant implications for the transformation of the content and for

the selection of an appropriate mathematical strategy for solving the problem. Gaeilgeoirí performed well on the process skills they employed, which is reflective of the pedagogic system in operation in Irish third level institutions.

Although this aspect of the project was only undertaken at third level education it is still a significant insight into the language practices employed by bilingual students in the Irish education system. *Both* languages are of importance to Gaeilgeoirí, yet the Irish third level institutions that participated in this study only teach through the medium of English with no facilitation for those learning mathematics through the medium of a second language (English). The Sapir (1949)-Whorf (1956) hypothesis was one of the primary theoretical lenses employed by the author when looking at the data. The basic premise of this hypothesis is that the vocabulary and phraseology of a particular language influences the thinking and perception of speakers of this language, and that conceptions not encoded in their language will not be available to them. A less severe form of this hypothesis has been employed by the author in that in that she supports the premise that language may not shape and determine our entire mathematical thinking, but that it may influence it to a certain degree (Sternberg, 2003). Clearly the Gaeilge language is still of importance to Gaeilgeoirí in this study even though they are engaged in learning mathematics through the medium of English. They rely on it for solving mathematical problems, for performing operations, for mathematical thinking and their conceptions of mathematics may be shaped by the language. Thus the importance of Gaeilge for mathematical learning and understanding cannot be ignored in the transition to English-medium mathematics education. The findings highlight the need for support measures to be introduced in order to cater for Gaeilgeoirí's linguistic and mathematical needs at third level education. Also, the model generated from the analysis of the qualitative data collected provides an analytical tool for further diverse areas of investigation within the Irish context.

Clearly Gaeilgeoirí face a number of challenges at the transition from Gaeilge-medium second level education to English-medium third level education. They face the challenge of recognising that they are bilingual and recognising the importance of their languages for mathematics learning and understanding. In order for Gaeilgeoirí to develop mathematical understanding, pedagogical practices are of key importance,

which in turn are evocative of cultural influences. Mathematics is a product of culture and diffused through pedagogy. Only by challenging the pedagogic and cultural practices employed at third level can we challenge Gaeilgeoirí's perceptions of mathematics and the type of mathematical understanding being developed. Gaeilgeoirí are presented with the challenge of developing mathematics understanding through the medium of English while drawing on their knowledge through the medium of Gaeilge. Thus Gaeilgeoirí need to recognise the importance of their languages for mathematical learning and understanding. Irish third level institutions are faced with the challenge of recognising that this cohort of learners exists and requires support structures to be implemented in order to counter the adverse effect learning through the medium of English is having on some of these students' mathematical understanding. Finally, and one of the most significant challenges Gaeilgeoirí are faced with is developing an awareness of their language use and its influence on mathematics learning and understanding. Gaeilgeoirí and mathematics educators need to recognise and be made aware of the significance of language for bilingual mathematics learners.

5.9 Conclusion

This Chapter presented the analysis and discussion of the findings from data collection undertaken in the preliminary and main study of this research project. It is clear that there are many interesting findings emerging from the data at both transitions in Irish education. These findings are summarised in the next Chapter with recommendations and conclusions drawn up for future work and research (Chapter 6).

6

Conclusion and Recommendations for Future Research

6.5 Introduction

This report investigated the influence of bilingualism on mathematics learning from an Irish perspective. The importance of language for the teaching, learning, understanding and communication of mathematics cannot be ignored. Educational objectives require students to understand mathematical concepts and to possess an ability to express their understanding of these concepts in written format (Rogan, 2005). However, the function of language does not lie solely in the representation of mathematical knowledge. Language is required for and engaged in bringing this knowledge into existence (Halliday & Martin, 1993). Furthermore, mathematics learners are required to possess competency both in everyday language and mathematics specific language, but competency in the natural language does not necessarily contribute to competency in the mathematics specific language (Lemke, 1990). Clearly, the intricate relationship between mathematics learning and a student's first language (or the first language of learning) is highly complex. This is further complicated when the language of instruction/learning changes, as is the situation faced by the majority of Gaeilgeoirí in Ireland. Addressing the needs of Gaeilgeoirí in the transition to English-medium mathematics education is paramount to this study. However, the first task is identifying their needs. Only then can we tackle these issues through pedagogic and supportive measures. The following sections will conclude the report and provide some recommendation.

6.6 Significant Overall Conclusions

The significant overall conclusions emerging from the research undertaken include:

- Language proficiency and performance on mathematical word problems are related for Gaeilgeoirí. The findings provide support for Cummins' Threshold

Hypothesis (1976) and demonstrate that bilingualism has a positive influence on mathematical learning once both languages (Gaeilge and English) are developed.

- This study is the first to investigate the nature of Additive and Subtractive bilingualism as influenced by school type attended from an Irish perspective. At both transitions support was found for Immersion Education and its influence on the development of Additive bilingualism and better performance on mathematical word problems. Maintenance Heritage Language education may contribute to Gaeilgeoirí experiencing Subtractive bilingualism on entering English-medium mathematics education.
- When assessed through the medium of English, Gaeilgeoirí in the transition from Gaeilge-medium primary level education to English-medium second level mathematics education experience a disadvantage of 8.7 percent in performance on mathematical word problems.
- Sources of difficulty encountered with the English mathematics register at both transitions include syntax, semantics and mathematics vocabulary.
- One of the key issues emerging from the qualitative research undertaken is the lack of awareness of bilingualism by Gaeilgeoirí and of the influence of language on mathematics teaching and learning.
- Gaeilgeoirí employ both languages when engaged in mathematical problem solving. This appears to be a subconscious process, ingrained in their process skills and provides confidence in tackling the mathematical problem presented.
- When engaged in mathematical word problem solving Gaeilgeoirí's understanding tends to break down at the comprehension stage. This in turn has significant implications for the transformation of the content and for the selection of an appropriate mathematical strategy for solving the word problem.
- For Gaeilgeoirí in the transition from primary (Gaeilge) to second level (English) mathematics education a significant relationship exists between their performance on the mathematical word problems through the medium of English and their Gaeilge language proficiency. Gaeilgeoirí with high proficiency in both languages, and those who were dominant in Gaeilge,

performed mathematically better than their monolingual peers. This suggests that learning mathematics through the medium of Gaeilge at primary level education may enhance mathematical understanding. Thus, future work should look at the Gaeilge mathematics register and how the Gaeilge language may facilitate mathematical understanding.

6.7 Recommendations

This study has generated some important insights into the bilingual mathematics situation that exists in Ireland. In this section the author proposes some recommendations arising from the key findings of the study.

6.7.1 Recommendations for Mathematics Teachers in English-medium Mathematics Education

The focus of this investigation has been on Gaeilgeoirí in the transition from Gaeilge-medium to English-medium mathematics education. Clearly the teacher is going to play a significant role in facilitating this transition. The following are a number of suggestions for teachers that can be incorporated into their pedagogic practices (Anstrom, 1999). We need to improve the quality of mainstream instruction so as to make language and mathematics content comprehensible for Gaeilgeoirí.

- Teachers need to ***make mathematics accessible*** and this can be achieved through introducing problem solving activities. By involving Gaeilgeoirí in solving interesting, real-life problems it will encourage critical thinking, in conjunction with basic skills development and practice (McLaughlin and McLeod, 1996).
- It is important ***to teach the language of mathematics***. Command of the English mathematics register will play an important role in the development of Gaeilgeoirí's mathematical ability (Coasaniti Dale and Cuevas, 1992).
- Mathematics teachers should ***create language supportive environments***. Planning classroom discourse that is inclusive of Gaeilgeoirí demands that teachers create mathematical environments and instructional situations that support students' linguistic and conceptual development (Bagley and Gallenberger, 1992).

- Mathematics teachers should try and ***connect the mathematics content to the students' background and experiences*** by addressing the cultural and educational background of Gaeilgeoirí (Buchanan and Helman, 1993).
- Teachers should ***vary instructional methods***. By doing so they will provide Gaeilgeoirí (and the other students) with an opportunity to learn in different ways, through individual, small group and whole class work (Buchanan and Helman, 1993).
- Finally, ***assessment should be authentic and meaningful*** (August and Pease-Alvarez, 1996). It may need to take place through the medium of English and in Gaeilge, depending on the language proficiency of the students. The test item should incorporate assessment of content knowledge and language proficiency so as to monitor Gaeilgeoirí mathematical and linguistic progress (August and Pease-Alvarez, 1996).

6.7.2 Recommendations for Mathematics Teachers in Gaeilge-medium Mathematics Education

Similarly, teachers involved in Gaeilge-medium primary and second level education can incorporate some aspects into their mathematics teaching so as to ease the transition to English-medium mathematics education for Gaeilgeoirí.

- Teachers in Gaeilge-medium schools need to ensure that their students are ***aware*** that they will be transferring to a new language of learning, either at second or third level education. In this sense, Gaeilgeoirí may be more prepared for the transition and it may not come as such a shock initially.
- It may be beneficial to introduce some ***partial instruction through the medium of English*** in the later years of primary and second level schooling.
- Assess upper primary and second level students' ***language proficiencies and mathematics performance*** so as to identify high ability Gaeilgeoirí who may excel at mathematics and Gaeilgeoirí who may experience difficulties with mathematics in the transition due to low proficiency in Gaeilge and in English.

6.7.3 Recommendations in relation to the Gaeilge Language

The research has demonstrated that the Gaeilge language and previous learning through the medium of Gaeilge is of importance to Gaeilgeoirí's mathematical learning through the medium of English. The author does not claim to be a linguistic expert but there are two key recommendations in relation to Gaeilge that she would like to pursue further that will provide additional significant insights into the Irish context and mathematics education. These are:

- It is apparent that Gaeilgeoirí rely on the Gaeilge language when engaged in mathematical problem solving through the medium of English. This suggests that '***code switching***' or language switching is occurring during mathematical problem solving for Gaeilgeoirí. Further research needs to look at this aspect and how it can be incorporated into the mathematics classroom so as to enhance the mathematics learning of Gaeilgeoirí immersed in English-medium mathematics education.
- There are cognitive benefits for learning mathematics through the medium of Gaeilge. Future Government initiatives may consider the option of providing students with the opportunity of studying particular subjects through the medium of Gaeilge at primary and post-primary level in English-medium schools. This may also assist in terms of language acquisition and national policies in relation to the Gaeilge language.

6.7.4 Recommendations for Pre-Service Teacher Education Programmes

Effectively preparing teachers to work with bilingual students requires a substantial shift in the way in which they are taught, and the content of the pre-service courses they take. Ideally all aspects of the coursework and teaching practice experiences would need to involve developing the skills and knowledge necessary for successful practice in bilingual classrooms. Is not sufficient to offer teachers just one or two modules within their normal courses. Thus the author would recommend the introduction of specific pre-service teacher (primary and second level) education courses targeted at teachers with a desire to work in Gaeltacht and Immersion schools in Ireland.

6.8 Future Research Directions

Bilingualism is no longer a rare occurrence in mathematics education (Barwell, Barton & Setati, 2007). However, what is rare is research conducted in an Irish mathematics education context involving both the Gaeilge and English languages. This investigation is a positive step towards examining the situation that exists in Ireland and assessing the learning needs and experiences of Gaeilgeoirí. The author has demonstrated that there is a need to continue with research in the area of bilingualism and its influence on mathematics education in an Irish context. Some suggestions for further research are outlined below:

- Further research is needed to both confirm the results emerging from this investigation with larger groups and in other locations, and also to investigate more fully issues of causality.
- There is a need to look further at the Gaeilge language and its influence on mathematics learning and understanding, in particular the influence of the use of 'code switching' or language switching by Gaeilgeoirí to help understand mathematical concepts.
- Research is needed on the mathematics register through the medium of Gaeilge and how this may influence mathematical learning.
- A specific teacher-training program should be introduced for primary and second level teachers of mathematics working in Gaeilge-medium education.
- There is an urgent need for further research in the area of Immersion Education (Gaelscoileanna/Gaelcholáistí) and Maintenance Heritage Language education (Gaeltacht schools) in all key subject areas as there is a clear lack of research undertaken to date.

6.9 Proposal for a Bilingual Primary and Second Level Education System

Initially, when the author was undertaking this study and researching the area under investigation, she had been in favour of studying through the medium of one's mother-tongue. However, after completing this study, the author feels that there are many benefits to be gained from becoming bilingual in both languages, as opposed to

dominance in one language. This in particular is evident from studies undertaken in Canada and Wales in which successful Immersion bilingual education programmes are in operation and students are reaping the cognitive benefits from these programmes (Cummins & Swain, 1986; Swain, 1996; Williams, 2002). Therefore the author recommends the development of language policies that place an emphasis on the development of *both* Gaeilge and English so that the students can become bilingual and thus reap the cognitive benefits of this. All Gaeilgeoirí should strive to achieve Additive bilingualism and accordingly the cognitive benefits from being bilingual.

6.10 Final Comment

The idea of this research came about from the author's own experience of transferring to English-medium third level education and her desire to highlight the issue so as to cater for those experiencing difficulties in the transition. What this investigation has demonstrated is that there are many issues (mathematical, language, cultural, pedagogical, and social) that Gaeilgeoirí have to cope with when confronted with a new language of learning and level of education. But what this investigation has also accentuated are the positive benefits that can be reaped from being bilingual. The task lies in implementing teaching interventions and support measures that will enhance these positive benefits for Gaeilgeoirí. Further, it is clear that:

> "The need for continuing the study can be justified not only in terms of equity and social justice, but also in terms of the richness of the research ground."
>
> (Gorgorió & Planas, 2001, p.31)

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